ECE 5650 Homework Set #1 Solved Typst Sample

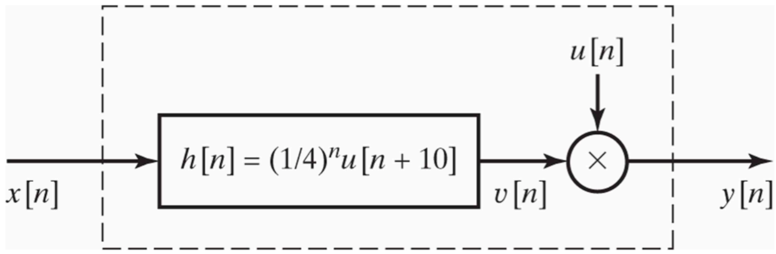
September 2, 2025

Mark Wickert

Contents

1. Custom Problem

Consider the system illustrated below. The output of an LTI system with an impulse response



$$h[n] = \left(\frac{1}{4}\right)^n u[n+10]$$

is multiplied by a unit step function $u[n]$ to yield the output of the overall system. Answer the following and briefly justify.

Figure 1: Text Problem 2.15 problem statement.

Part a

Is the overall system LTI?

- The system is linear from the input $x[n]$ to the interstage out $v[n]$
- When $v[n]$ is multiplied by the step function $u[n]$, the output becomes time varying since signals for $n < 0$ are not passed while those for $n \geq 0$ are passed
- If the input signal is delayed or advanced in time, the output $v[n]$ will be turned on/off differently due to the gating action of $u[n]$
- We conclude that the system is not LTI

Part b

Is the overall system causal?

- The impulse response from $x[n]$ to $v[n]$ is not causal by virtue of the fact that it is not zero for $n < 0$
- The impulse response turns on at $n = -10$
- We conclude that the system is not causal

Part c

Is the overall system stable in the BIBO sense?

- First see if the impulse response $h[n]$ satisfies the LTI BIBO stability theorem

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-10}^{\infty} |\frac{1}{4}|^n = \sum_{k=0}^{\infty} |\frac{1}{4}|^{k-10} \tag{1}$$

$$= 4^{10} \sum_{k=0}^{\infty} \frac{1}{1-\frac{1}{4}} = \frac{4^{10}}{1-\frac{1}{4}} < \infty \tag{2}$$

Hence BIBO stable.

- Now since $h[n]$ BIBO stable and clearly the gating function is bounded, i.e.$|u[n]| < \infty$, we conclude that the overall system is BIBO stable

2. Text 2.20

Consider the difference equation representing a causal LTI system

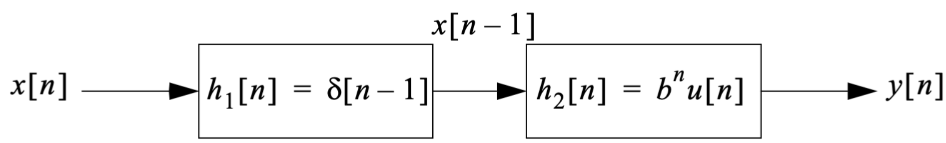
$$y[n] + \frac{1}{a}y[n-1] = x[n-1]$$

Figure 2: Text Problem 2.20 problem statement.

Part a

Find the impulse response of the system, $h[n]$, as a function of constant a.

- There are many approaches to working this problem
- The classical solution to LCCDEs comes to mind first of all
- **Cascade Approach**: Another approach is to view the system as a cascade of two systems, one a simple one sample delay and the other the familiar LCCDE having impulse response $b^n u[n]$ which corresponds to $y[n] = by[n-1] + x[n]$
- We form a cascade below:



- The impulse response of the cascade is the convolution of the two impulse responses, thus

$$h[n] = \delta[n-1] * b^n u[n] = b^{n-1} u[n-1]$$

- Here we recognize that $b = -1/a$, so finally

$$h[n] = \left(-\frac{1}{a}\right)^{n-1} u[n-1]$$

Figure 3: Text Problem 2.20 part a cascade solution.

Part b

For what range of values of a will the system be stable?

- The stability of the system requires that the ouput be bounded for a bounded input
- For an LTI system the impulse response must satisfy

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

$$= \sum_{n=1}^{\infty} \left|-\frac{1}{a}\right|^{n-1} = \sum_{n=0}^{\infty} \left|-\frac{1}{a}\right|^{n} = \frac{1}{1-|1/a|} = \frac{|a|}{|a|-1} < \infty$$

- Thus we must have $|a| > 1$

Figure 4: Text Problem 2.20 part b solution

3. Text 2.23a & c

For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time invariant.

Part a

Poor quality scan. Smudgy hand writing due to erasures. Be careful.

$$T(x[n]) = (\cos \pi n)\, x[n]$$

- This system can be viewed as a simple memoryless system with time varying gain $g[n] = \cos(\pi n)$
  - The system is stable since the gain coefficient is bounded, i.e., $|g[n]| < \infty$
  - The system is causal since the present output depends only on the present input
  - The system is linear since the system output is a scaled version of the input
  - The system is not time invariant since the gain coefficient is not a constant, rather a function of time

Figure 5: Text Problem 2.23a solution

Part c

Fade out near the end. Be careful.

$$T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n-k]$$

- THIS SYSTEM IS THE PRODUCT OF TWO WAVEFORMS $x[n]$, THE INPUT AND a gain scaling OR SWITCHING waveform THAT TURNS ON AT $n=0$
- NOTE SINCE

$$\sum_{k=0}^{\infty} \delta[n-k] = u[n]$$

WE CAN ALSO VIEW THIS SYSTEM AS $T(x[n]) = x[n]\,u[n]$

- The SYSTEM IS STABLE AS THE OUTPUT IS EITHER $x[0]$ or 0
- The SYSTEM IS CAUSAL SINCE THE OUTPUT IS JUST A SCALED VERSION OF THE INPUT
- The SYSTEM IS linear SINCE THE OUTPUT IS JUST A scaled VERSION OF THE INPUT
- The SYSTEM IS NOT TIME INVARIANT SINCE THE CONSTANT IS TIME VARYING

Figure 6: Text Problem 2.23c solution

4. Text 2.29

An LTI system has impulse response defined by

$$h[n] = \begin{cases} 0, & n < 0 \\ 1, & n = 0,\ 1,\ 2,\ 3 \\ -2, & n = 4,\ 5 \\ 0, & n > 5 \end{cases} \tag{3}$$

Determine and plot the output $y[n]$ in response to input $x[n]$ as listed in the parts below.

- To begin with we can write $h[n]$ in a functional form as

$$h[n] = u[n] - 3u[n-3] + 2u[n-6] \tag{4}$$

Plotted below using wxMaxima and then Python.

- Plot the impulse response using wXMaxima

```
 -->  n_p29:makelist(n,n,-2,20);
(n_p29) [-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]
 -->  h_p29:dstep(n_p29,0) - 3*dstep(n_p29,4) + 2*dstep(n_p29,6)$
 -->  stemlist(h_p29,[],[],"Impulse Response h[n]",[],-2);
```

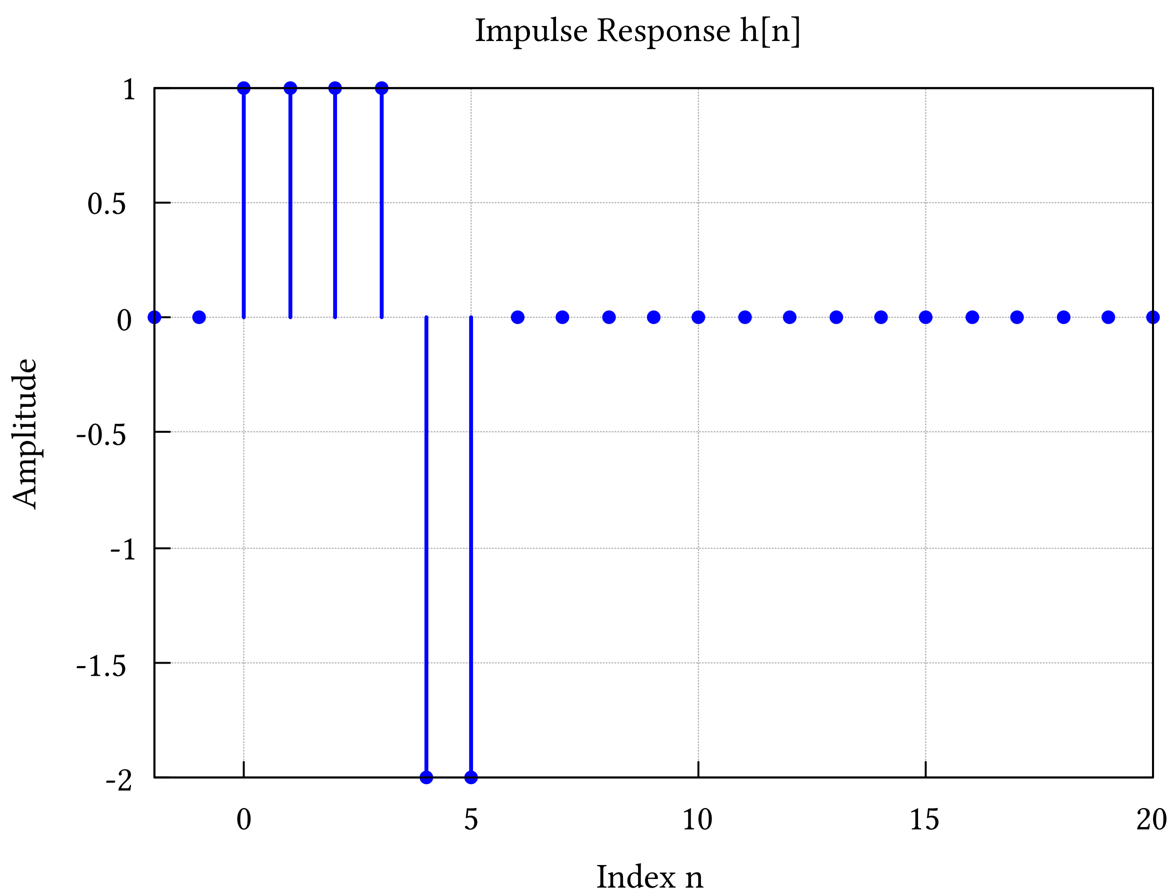


Figure 7: Text Problem 2.29 defining the $h[n]$.

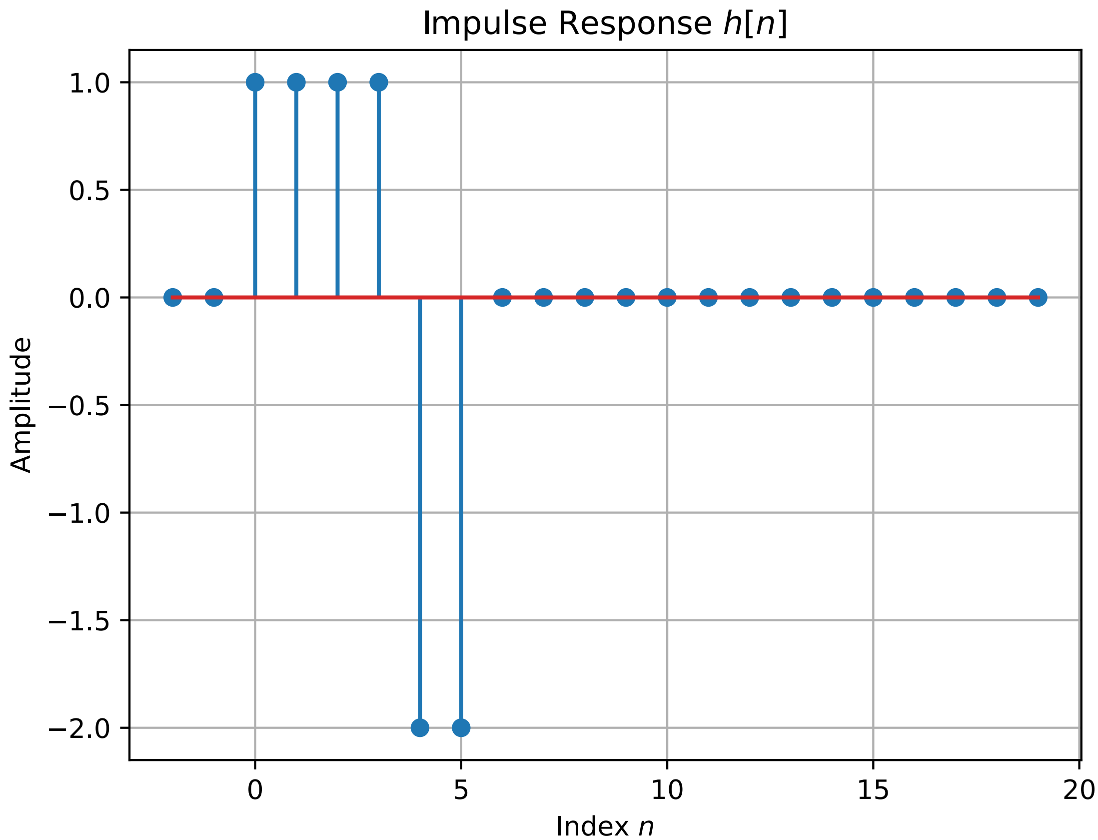
- Plot the impulse response using Python in Jupyter. This import is directly from the notebook using Callisto
- First we show the imports cell

In [1]:
```
from numpy import *
from matplotlib.pyplot import *
import sk_dsp_comm.sigsys as ss
import fxpmath as fxp
import sk_dsp_comm.fir_design_helper as fir_d
import scipy.signal as signal
from IPython.display import Audio, display
from IPython.display import Image, SVG

%config InlineBackend.figure_formats=['svg'] # SVG inline viewing
```

- Next show the plot

In [4]:
```
n_p29 = arange(-2,20)
h_p29 = ss.dstep(n_p29) - 3*ss.dstep(n_p29-4) + 2*ss.dstep(n_p29-6)
stem(n_p29,h_p29)
title(r'Impulse Response $h[n]$')
xlabel(r'Index $n$')
ylabel(r'Amplitude')
grid();
# savefig('Text_2-29_Imp_Resp.svg')
```



Part a

$x[n] = u[n]$

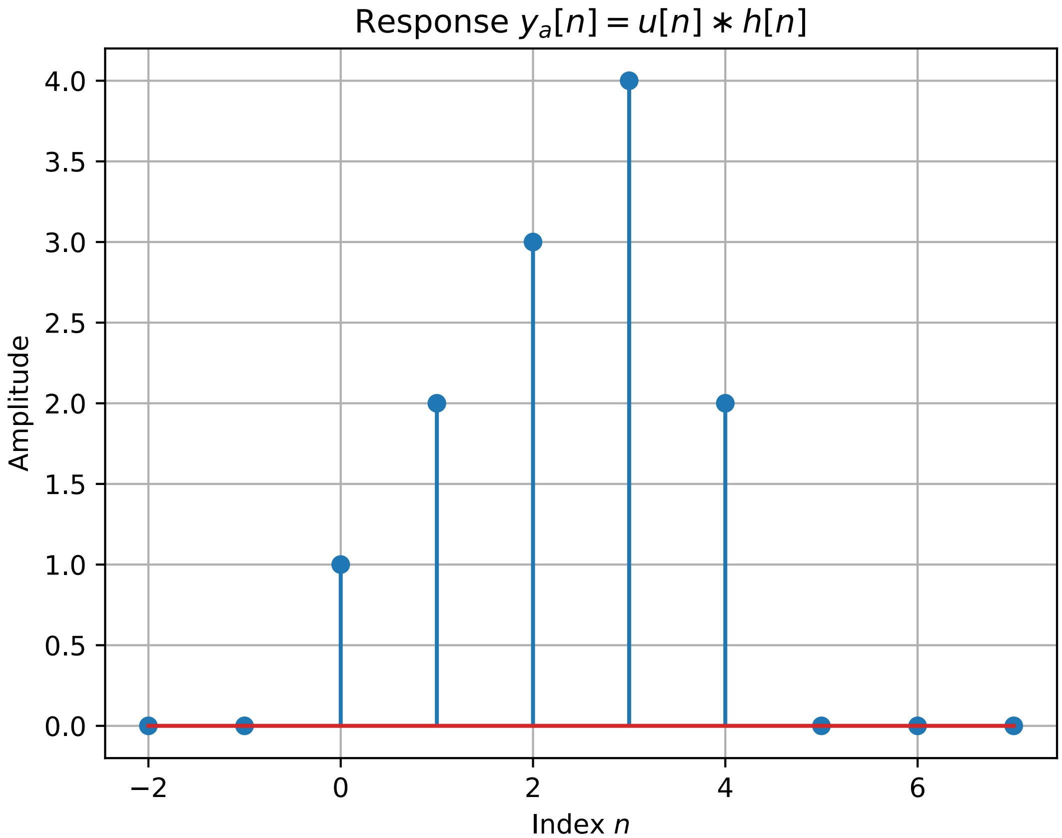
- Here we will use directly take `markdown` cells from the notebook that, also contain math equations, and directly place in the Typst document

(i) Direct Simulation Here we use the direct respresentation of the impulse response given as

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - 2\delta[n-5] \quad (5)$$

- Show the plot from the notebook

In [16]:
```
n = arange(-2,8)
y_a = signal.lfilter([1,1,1,1,-2,-2],1,ss.dstep(n))
stem(n,y_a)
title(r'Response $y_a[n] = u[n] \ast h[n]$')
xlabel(r'Index $n$')
ylabel(r'Amplitude')
grid();
```



- Finally the full analytical response using the mathmatics directly from the Jupyter notebook

(ii) An Analytical Approach We start by writing

$$y_a[n] = u[n] * h[n] = u[n] * \{u[n] - 3u[n-4] + 2u[n-6]\} \tag{6}$$

Distribute into three convolutions superimposed. We need to develop a general convolution of the form

$$u[n] * u[n-n_0] = \sum_{k=-\infty}^{\infty} u[n-k] \cdot u[k-n_0] = \begin{cases} 0, & n < n_0 \\ \sum_{k=n_0}^{n} 1 \cdot 1 = n - n_0 + 1, & n >= n_0 \end{cases} \tag{7}$$

So now using the above result we just have to super impose the individual responses

$$y_a[n] = (n+1)u[n] - 3(n-4+1)u[n-4] + 2(n-6+1)u[n-6] \tag{8}$$

or reducing

$$y_a[n] = \begin{cases} n+1, 0 \leq n \leq 3 \\ 2, n = 4 \\ 0, \text{otherwise} \end{cases} \tag{9}$$

A. Appendix

If needed we can have an appendix.

```
# A code cell with code
n = arange(-5,10)
st = ss.dstep(n)
# x = ss.dstep(n) - ss.dstep(n-1)
delta = diff(st)
# x = ss.dimpulse(n+1)
stem(n[1:],delta)
ylim([-.1,1.1])
title(r'Plotting $\delta[n]$')
grid();
savefig('my_impulse.svg')
```