Chapter 6

Advanced Data Communications Topics (text chapter 10)

Contents

6.1 Introduction

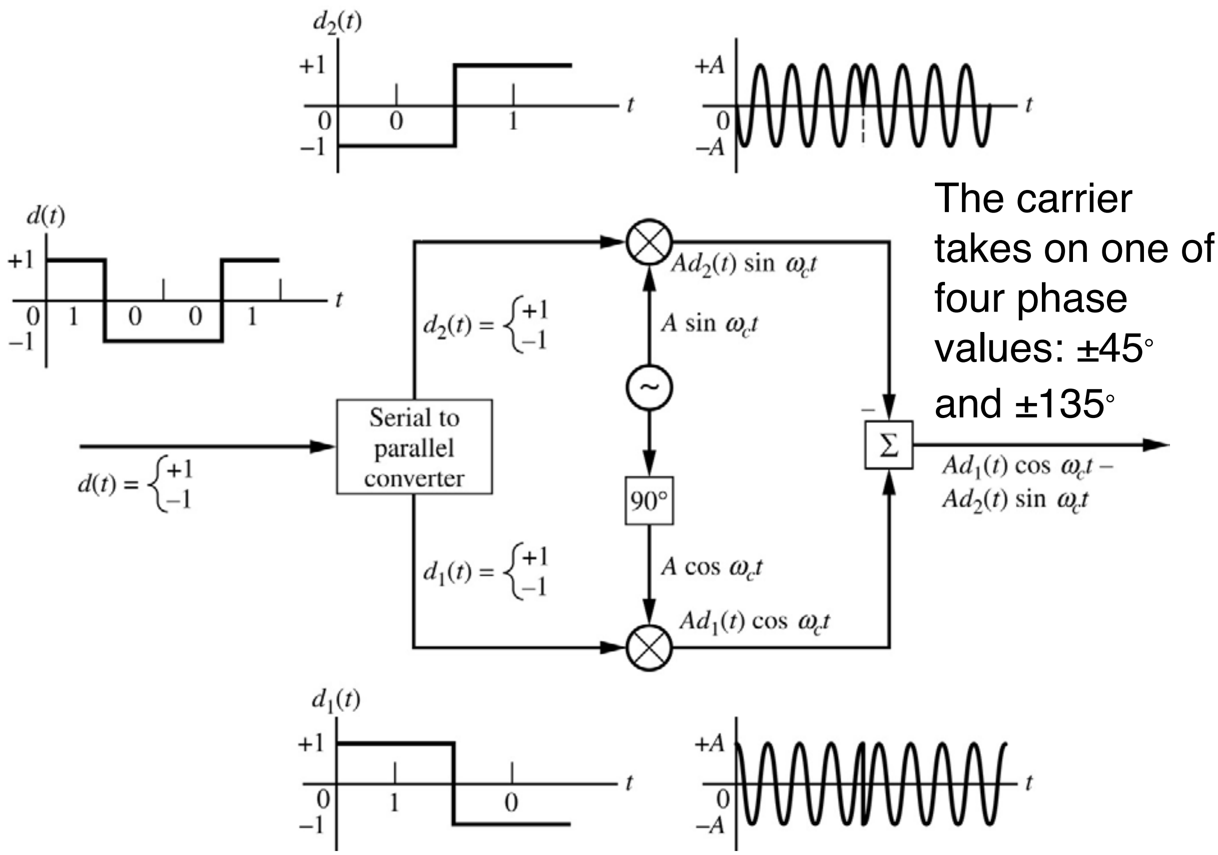
The previous chapter dealt almost exclusively with binary data communications. The is chapter begins with an investigation of M-ary schemes beyond M-ary PAM. Many other digital communications topics are also addressed. In fact, this chapter of Z&T packed with introductions to a vast array of topics: power efficiency, bandwidth efficiency, synchronization, spread-spectrum, multicarrier modulation (MCM) and orthogonal frequency division multiplexing (OFDM), and cellular radio communications, including LTE.

6.2 M-ARY Data Comm Systems

From this point forward we will need to distinguish between the symbol period, T_s, and the bit period T_b. With M-ary signaling we send b-bits of information per symbol. In general $M \geq 2$.

6.2.1 M-ary Schemes Based on Quadrature Multiplexing

- Quadrature multiplexing (QM) is first introduced in Comm I (Z& T p.206)
- It has been alluded since we first discussed the complex baseband representation of bandpass signals
- A quadrature multiplexing modulator configured to generate *quadriphase-shift keying* (QPSK) is shown below:



Quadrature modulator for the generation of QPSK

- Notice that the serial data stream is first demultiplexed into two parallel streams, i.e., even and odd bit sequences

- In a more detailed architecture or at least one that employs pulse shaping, the output of the 1:2 demux would drive a pulse shaping filter via an impulse train modulation

- The an I and Q baseband waveforms are shifted up to the desired carrier location via cos( ) and sin( ) multipliers

$$
\begin{aligned}
x_c(t) &= A\big[m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)\big] \\
&\triangleq R(t)\cos\big[2\pi f_c t + \theta_i(t)\big] \\
&= A\big[x_I(t)\cos(2\pi f_c t) - x_Q(t)\sin(2\pi f_c t)\big]
\end{aligned}
$$

Note: $R(t) = |x_I(t) - jx_Q(t)|$ and $\theta_i(t) = \angle[x_I(t) - jx_Q(t)]$

- Because of the orthogonality between the in-phase and quadrature signals paths (under ideal conditions), QM is like having two separate modulation schemes operating in parallel
  - In practical applications, this fact is taken advantage of

QPSK

- With QPSK we have $m_1(t) = d_1(t)$ and $m_2(t) = -d_2(t)$
- Assuming independent bit errors between the two carriers the probability of a correct symbol must be

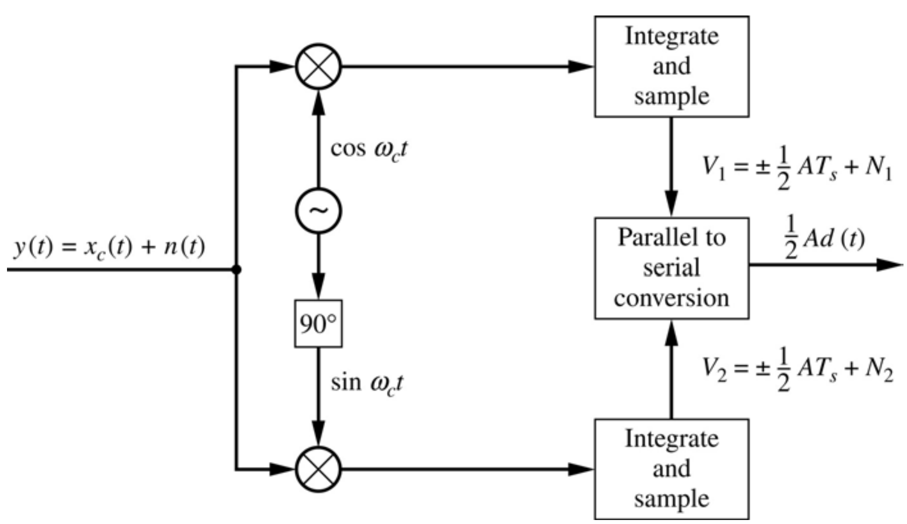
$$P_c = (1 - P_{E_1})(1 - P_{E_2}),$$

so

$$P_s = 1 - (1 - P_{E_1})(1 - P_{E_2})$$

where P_{E_1} and P_{E_2} are the bit error probabilities of each I/Q channel and P_s is the symbol error probability

- To find P_{E_1} and P_{E_2} we consider a rectangular pulse shape demodulator



I/Q demodulator for rectangular pulse shape QPSK in AWGN

- With

$$y(t) = A\big[d_1(t)\cos(2\pi f_c t) - d_2(t)\sin(2\pi f_c t)\big] + n(t)$$

it follows that the decision statistics V_1 and V_2 are of the form

$$\begin{aligned} V_1 &= \int_0^{T_s} y(t)\cos(2\pi f_c t)\,dt = \pm\frac{1}{2}AT_s + N_1 \\ V_2 &= \int_0^{T_s} y(t)\sin(2\pi f_c t)\,dt = \pm\frac{1}{2}AT_s + N_2 \end{aligned}$$

where

$$\begin{aligned} N_1 &= \int_0^{T_s} n(t)\cos(2\pi f_c t)\,dt \\ N_2 &= \int_0^{T_s} n(t)\sin(2\pi f_c t)\,dt \end{aligned}$$

- We know from the narrowband noise modeling that N_1 and N_2 should be uncorrelated (text homework problem) Gaussian RV, hence they are independent and V_1 and V_2 are also independent

- The mean of N_1 is zero and the variance is

$$\begin{aligned} \sigma_1^2 &= E\left[\left(\int_0^{T_s} n(t)\cos(2\pi f_c t)\,dt\right)^2\right] \\ &= \int_0^{T_s}\int_0^{T_s} \underbrace{E[n(t)n(\lambda)]}_{\delta(t-\lambda)} \cos(2\pi f_c t)\cos(2\pi f_c \lambda)\,dt\,d\lambda \\ &= \frac{N_0}{2}\int_0^{T_s} \cos^2(2\pi f_c t)\,dt \\ &= \frac{N_0 T_s}{4} \end{aligned}$$

- As in the case of equally likely binary antipodal signaling,

$$\begin{aligned} P_{E_1} &= P[d_1 = 1]P[E_1|d_1 = 1] \\ &\quad + P[d_1 = -1]P[E_1|d_1 = -1] \\ &= P[E|d_1 = \pm 1] \end{aligned}$$

- Using the variance calculation and the known mean value of V_1, we have

$$\begin{aligned} P[E_1|d_1 = 1] &= \int_{-\infty}^{-AT_s/2} \frac{\exp[-n_1^2/(2\sigma_1^2)}{\sqrt{2\pi\sigma_1^2}}\, dn_1 \\ &= Q\left(\sqrt{\frac{A^2 T_s}{N_0}}\right) \end{aligned}$$

and the same result holds for P_{E_2}

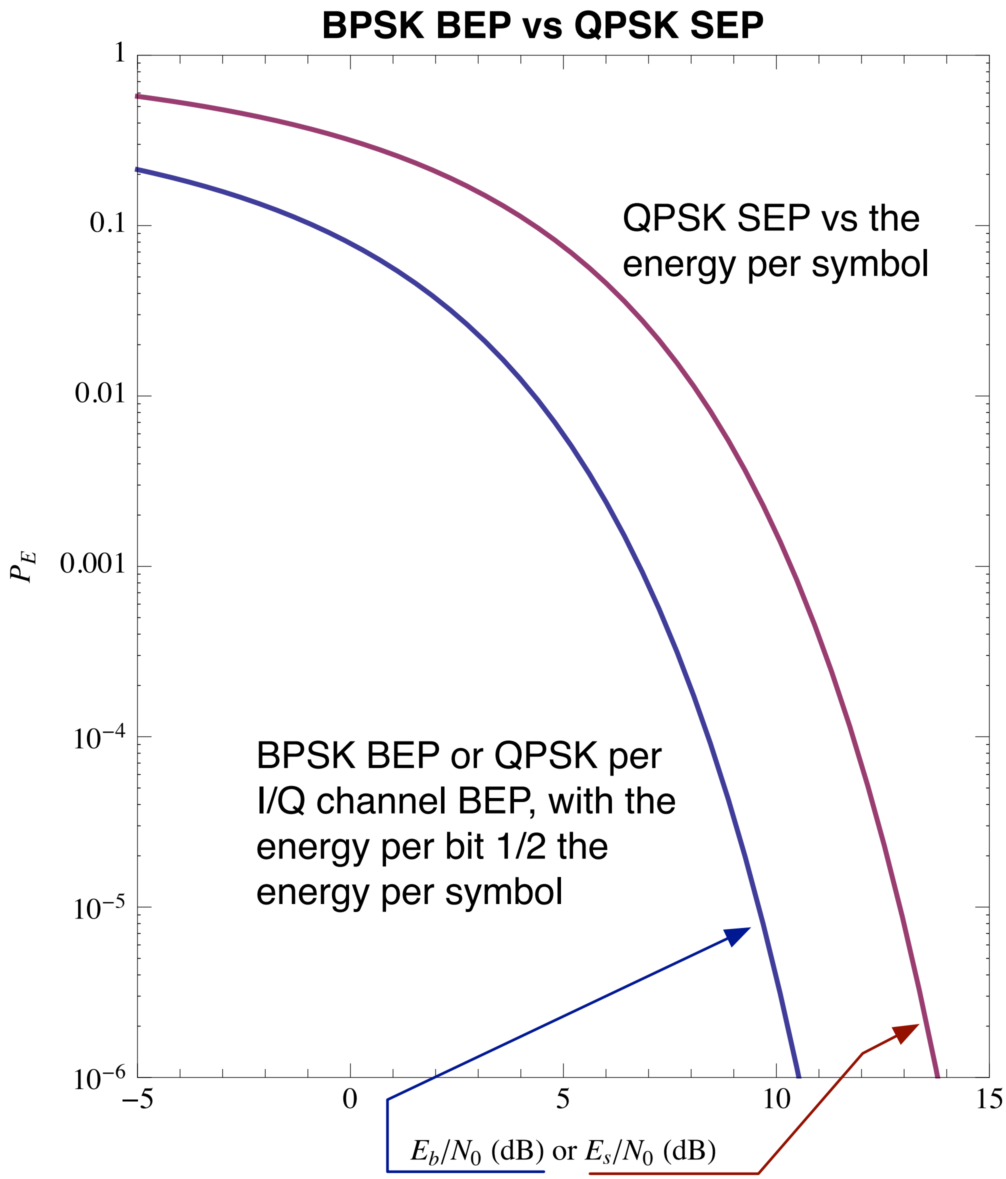
- Putting this together, the symbol error probability is

$$\begin{aligned} P_E &= P_s = 1 - P_c \\ &= 1 - (1 - P_{E_1})^2 \simeq 2P_{E_1},\ P_{E_1} \ll 1 \end{aligned}$$

or writing out in terms of the Q-function

$$P_E \simeq 2Q\left(\sqrt{\frac{A^2 T_s}{N_0}}\right) = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

Note: The average energy per symbol is $A^2 T_s$



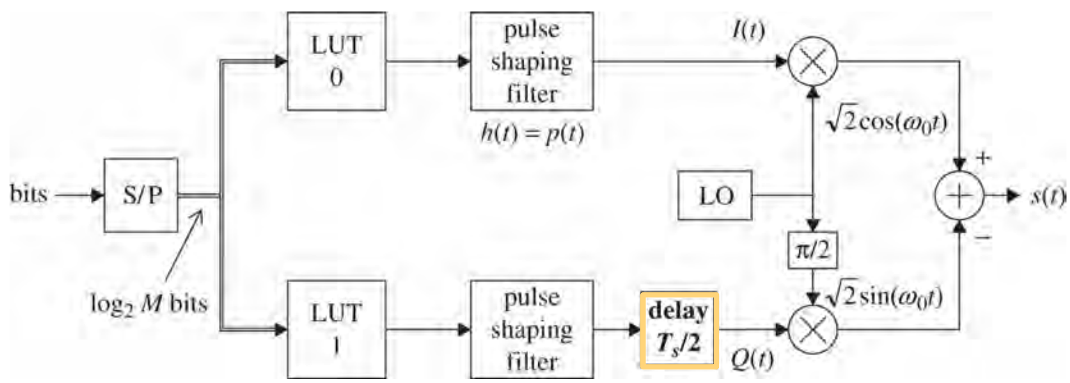
Comparing BPSK BEP and QPSK SEP

6.2.2 OQPSK Systems

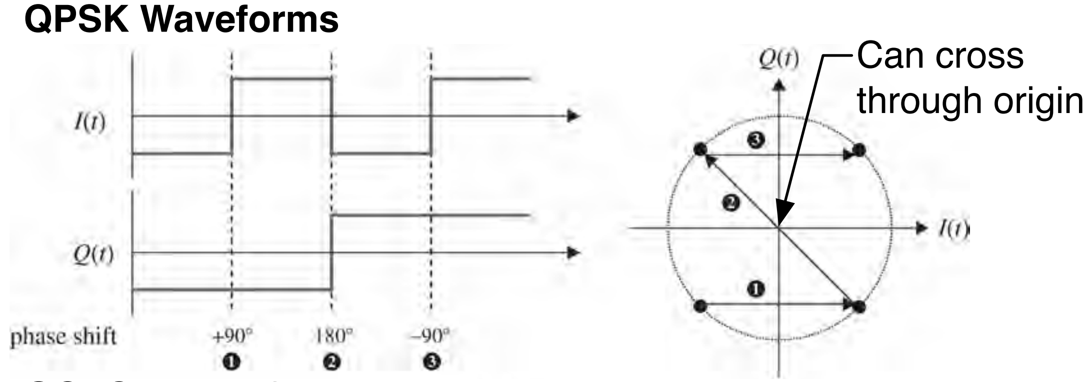
- The QPSK waveform design is such that both waveforms $d_1(t)$ and $d_2(t)$ switch at the same time

- With rectangular pulse shaping, this means the carrier phase may abruptly change by $\pm 180°$

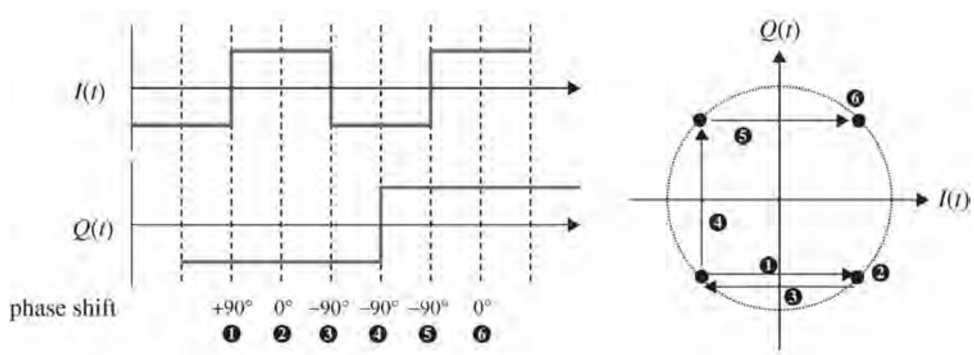
OQPSK Modulator



QPSK Waveforms

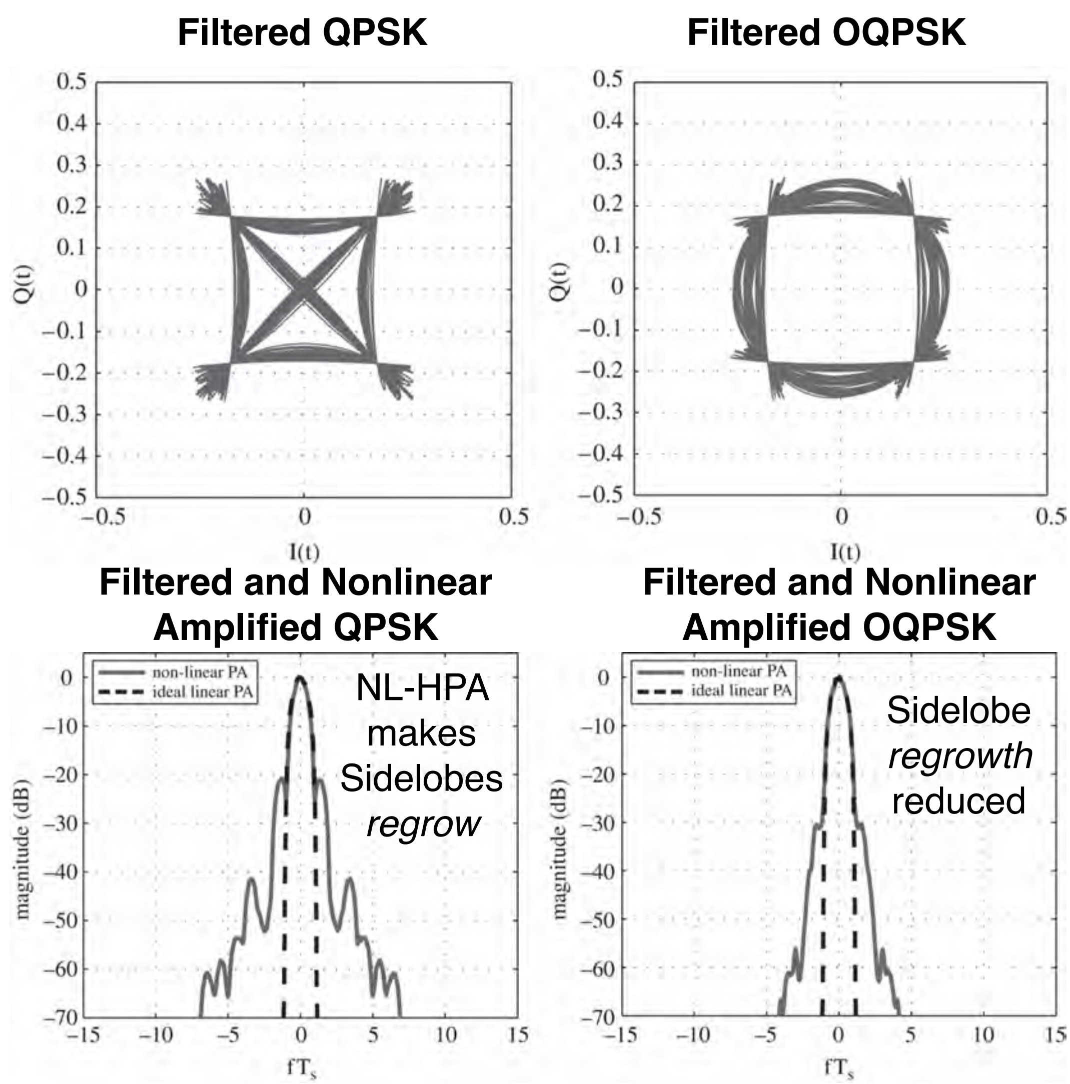


OQPSK Waveforms



OQPSK waveform generation compared with QPSK

- When a QPSK signal is then bandpass filtered large envelope deviations occur, making the signal more susceptible to *spectral sidelobe regrowth* when amplified by a nonlinear high power amplifier (NL-HPA)

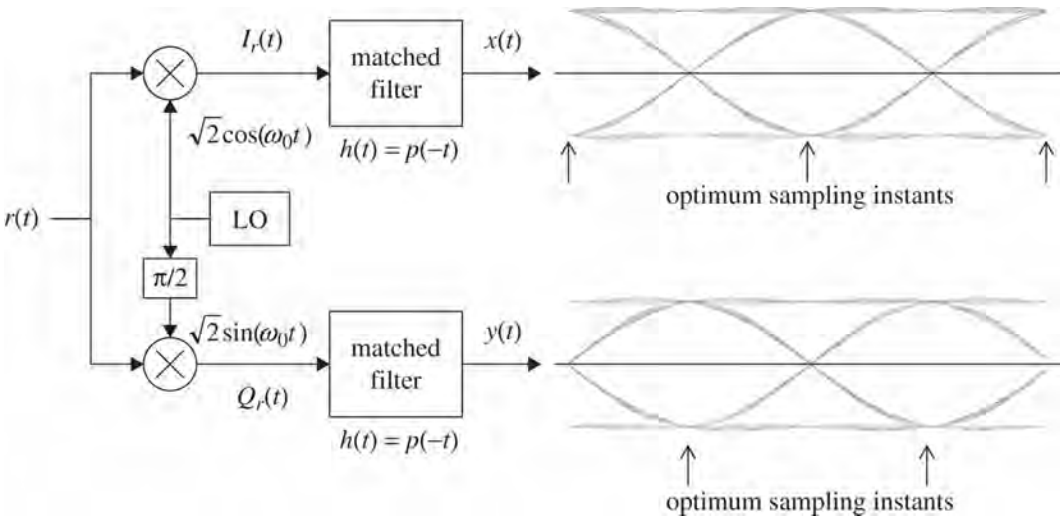


Compare QPSK and OQPSK with filtering and NL-HPA

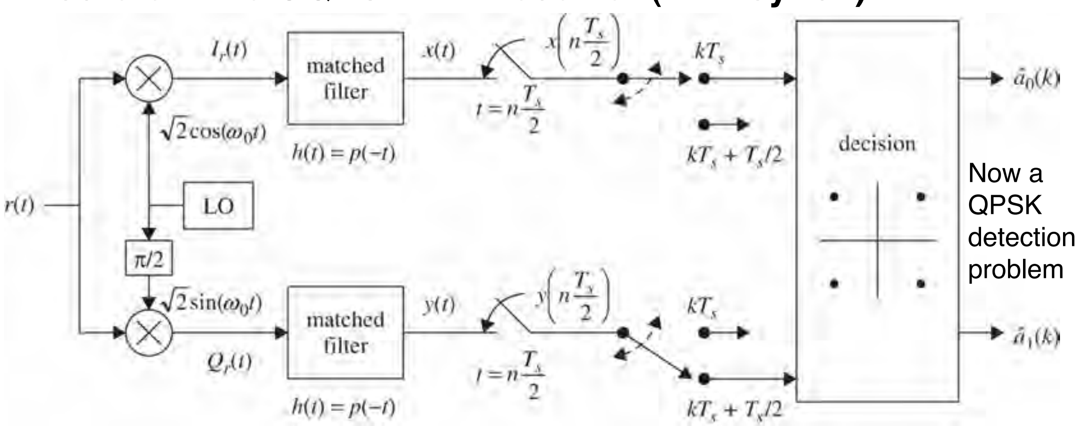
- At the receiver minimal implementation changes are required compared to that of QPSK

- In satellite communication systems OQPSK, also known as staggered-QPSK (SQPSK), is quite popular

Continuous-Time OQPSK MF Receiver



Discrete-Time OQPSK MF Receiver (with synch)



OQPSK receiver

- The error probability performance of OQPSK is identical to that of QPSK

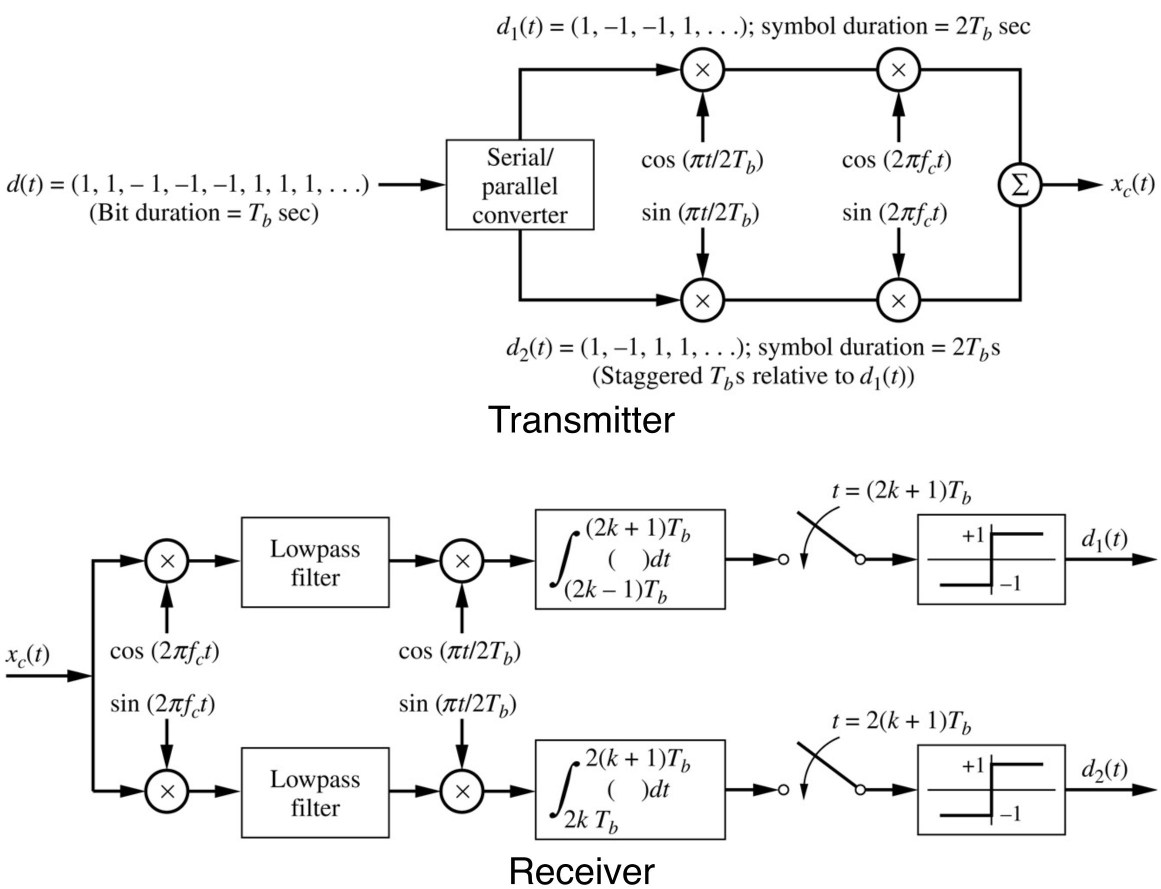
- A constraint imposed by OQPSK is that the bit rate of the parallel data streams must be the same, to insure that phase changes of no more that $\pm 90°$ are maintained

6.2.3 MSK Systems

- Other variations in the choice of $d_1(t)$ and $d_2(t)$ are possible
- In particular, a scheme known as *minimum shift-keying* can be generated by letting the original $m_1(t)$ and $m_2(t)$ signals be defined as

$$m_1(t) = d_1(t)\cos(2\pi f_1 t)$$
$$m_2(t) = d_2(t)\sin(2\pi f_1 t),$$

  where $d_1(t)$ and $d_2(t)$ are rectangular pulse shape data signals having ± 1 amplitudes which switch at $T_s = 2T_b$ intervals, and f_1 may for example be $1/(2T_s) = 1/(4T_b)$



MSK Type I and II transceiver block diagram

- It can be shown via trigonometry, that we may write

$$x_c(t) = A\cos\left[2\pi f_c t + \theta_i(t)\right]$$

where

$$\theta_i(t) = \tan^{-1}\left[\frac{d_2(t)}{d_1(t)}\tan(2\pi f_1 t)\right]$$

- It can be noted that if consecutive data bits (serial stream) are both 1 or both -1, then

$$\theta_i(t) = 2\pi f_1 t$$

and if data bits are different

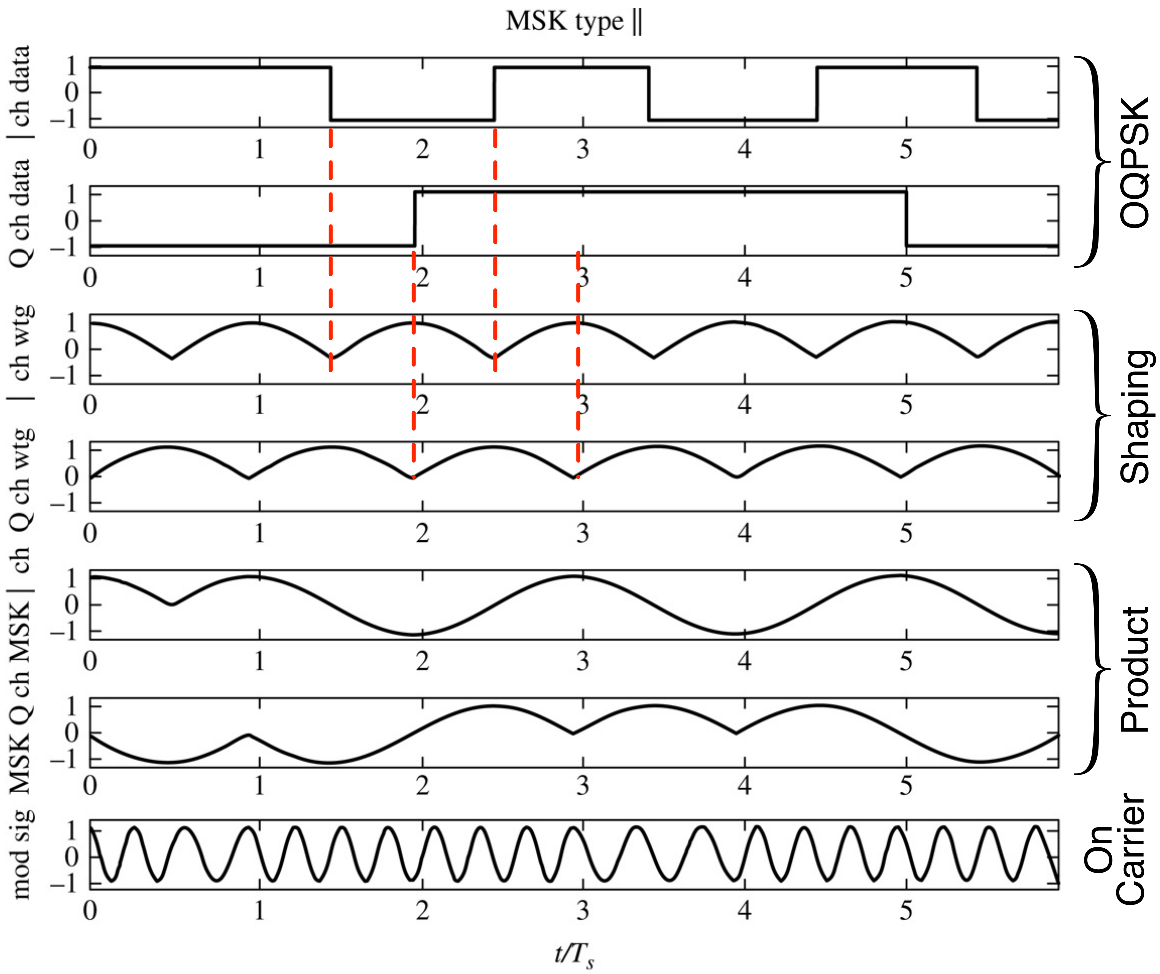
$$\theta_i(t) = -2\pi f_1 t$$

- Taking $1/(2T_s) = 1/(4T_b)$, we have *Type I MSK* if the sin( ) and cos( ) weighting functions originally described are invoked (note these functions themselves alternate in sign)

- We have *Type II MSK* if the sin( ) and cos( ) weighting functions are always taken as positive half cycle sin/cos pulses

- In actuality Type II MSK is like OQPSK with half sine pulse shaping over each of the data bits of duration T_s on the I and Q channels (used in high-rate IEEE 802.15.4 Zigbee)

- We can also view MSK as a form of frequency-shift keying, actually *fast frquency-shift keying* (FFSK) or also as binary *continuous phase modulation* (CPM)



MSK Type II waveforms as half-sine shaped OQPSK

- The error probability analysis of MSK is identical to QPSK, so the performace is the same
- We will see later that the power spectrum spectrum of MSK is different, but recall the half-sine pulse shape for a random binary pulse train

Continuous Phase Modulation (CPM)

- Understanding the CPM viewpoint leads to *Gaussian minimum-shift keying* (GMSK), which is the subject of the next section

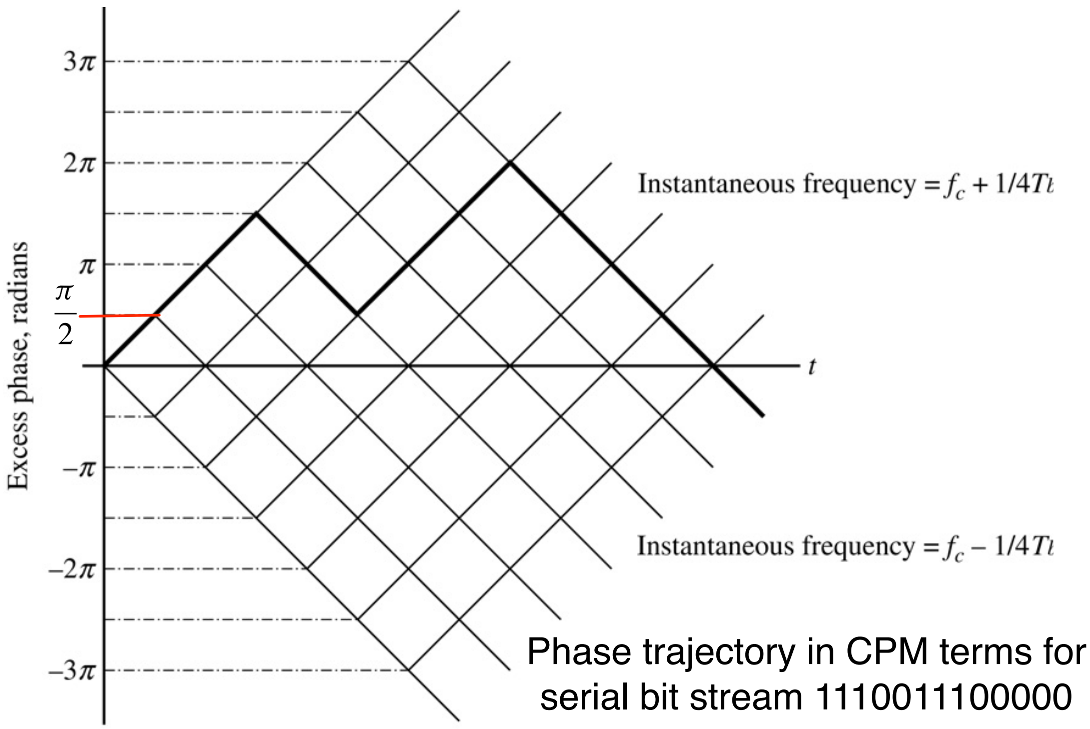
- In complex baseband terms, a binary CPM signal takes the form
$$s(t,\boldsymbol{\alpha}) = \sqrt{\frac{E_b}{T_b}} \exp\big(j\phi(t,\boldsymbol{\alpha})\big)$$
where E_b is the bit energy, T_b is the bit duration ($R_b = 1/T_b$ is the bit rate), $\boldsymbol{\alpha} = \{\alpha_i\}$ are the information symbols, here ± 1, and $\phi(t,\boldsymbol{\alpha})$ is the underlying phase deviation
$$\phi(t) = 2\pi \sum_{i=-\infty}^{n} \alpha_i h q(t - iT_b),\ nT_b \leq t \leq (n+1)T_b,$$
where h is the modulation index (typically rational, e.g., 1/2 for MSK)

- $q(t)$ is the modulation phase pulse, which is related to the underlying frequency pulse shape $g(t)$ via
$$q(t) = \int_0^t g(\sigma)\, d\sigma,$$
with the requirement that $\int_0^{T_b} g(\sigma)\, d\sigma = 1/2$

- For MSK $g(t)$ is a rectangle pulse shape of duration T_b, thus $q(t)$ is a phase ramp, which makes the phase trajectory taken by MSK linear segments that change by $\pm\pi/2$ radians each bit duration

- The exact phase trajectories taken by MSK I & II, and this form based on CPM, are different, but the main theme is that the carrier phase moves linearly by $\pi/2$ in all cases

- The *Phase trellis* is an instructive way to view MSK



Phase trellis for binary CPM with rect frequency pulses (MSK)

Gaussian MSK (GMSK)

- The CPM view of MSK requires that the frequency pulse be rectangular of duration T_b

- To smooth the phase transitions further, other pulse shapes, $g(t)$, may be chosen, again subject to the constraint that

$$\int_0^{T_b} g(\sigma)\, d\sigma = 1/2$$

- An alternate modulator for the CPM view of MSK is to send ± 1 NRZ data into an FM modulator, with deviation constant such that the carrier phase rotates $\pm\pi/2$ over each T_b interval

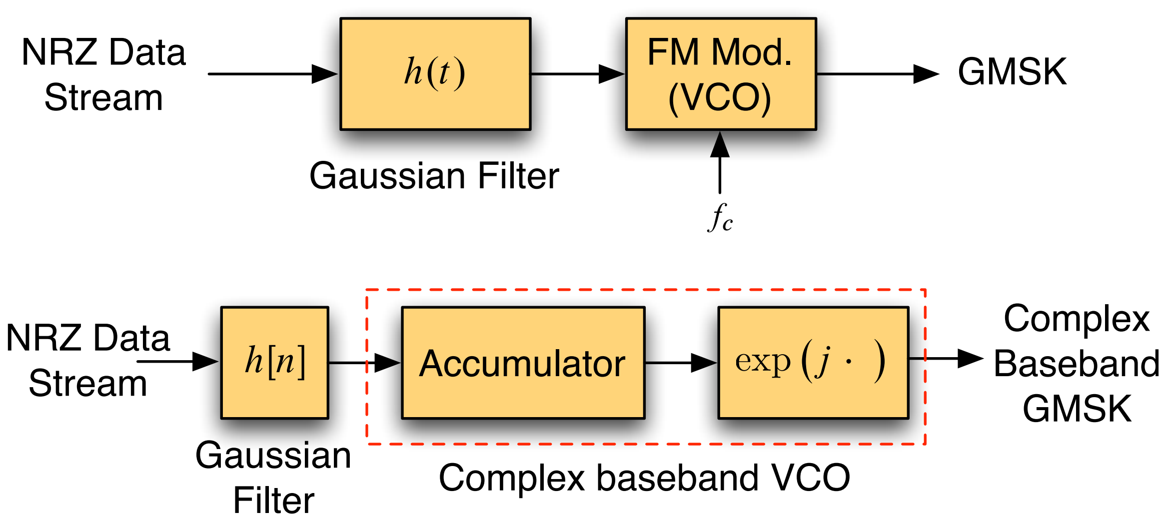
- Under GMSK we add a Gaussian shaped lowpass filter between the NRZ data stream and the FM modulator (VCO)

- The desired filter frequency response and corresponding impulse response are

$$H(f) = \exp\left[-\frac{\ln 2}{2}\left(\frac{f}{B}\right)^2\right]$$

$$h(t) = \sqrt{\frac{2\pi}{\ln 2}}B\exp\left(-\frac{2\pi^2 B^2}{\ln 2}t^2\right)$$

where B is the two-sided 3db bandwidth of the filter



GMSK generation

- The frequency pulse in this case involves integrating the Gaussian pdf, so it is a defined in terms of the error function

$$\begin{aligned} g(t) &= \int_{-\infty}^{t+T_b/2} h(\tau)\,d\tau - \int_{-\infty}^{t-T_b/2} h(\tau)\,d\tau \\ &= \frac{1}{2}\left\{\text{erf}\left[\sqrt{\frac{2}{\ln 2}}\pi BT_b\left(\frac{t}{T_b}+\frac{1}{2}\right)\right]\right. \\ &\quad \left. + \text{erf}\left[-\sqrt{\frac{2}{\ln 2}}\pi BT_b\left(\frac{t}{T_b}-\frac{1}{2}\right)\right]\right\} \end{aligned}$$

where $\text{erf}(u) = 2/\sqrt{\pi}\cdot\int_0^u \exp(-t^2)\,dt$ is the error function

- The product BT_b is a design parameter which controls the spectral occupancy of GMSK and also the ISI present in $s(t)$ (think receiver BEP degradation)

Example 6.1: GMSK MATLAB Simulation

- A complex baseband modulator was written to generate GMSK using a truncated Gaussian impulse response

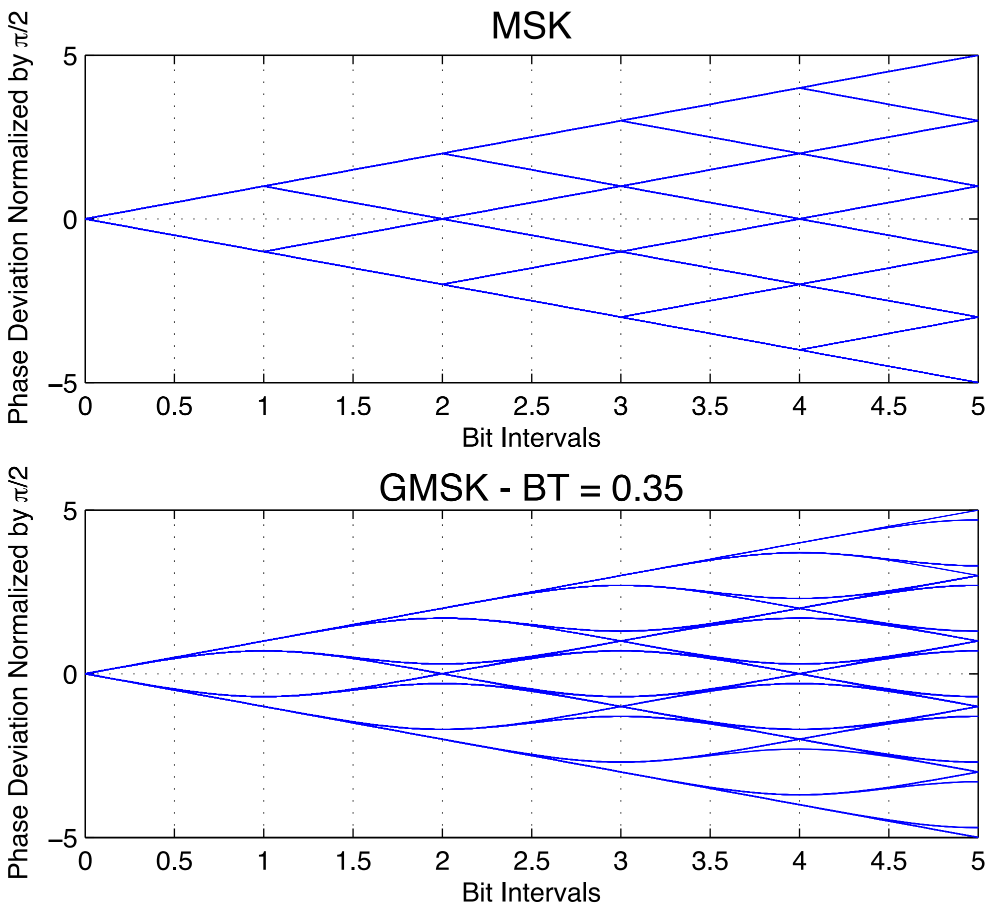
```
%///////////////////////////////////////////////////////////////////
function [x,data] = gmsk(BT, K, Ns, MSK, in_data)
%///////////////////////////////////////////////////////////////////
%         [x,data] = gmsk(BT, K, Ns, MSK, in_data)
%  BT  = premodulation Bb*T product which sets
%        the bandwidth of the Gaussian lowpass filter
%  K   = number of symbols processed
%  Ns  = the number of samples per bit
%  MSK = 0 for no shaping which is standard MSK,
%        MSK <> 0 --> GMSK is generated.

if nargin  == 5
    [data, y] = m_ary(2,K,Ns,in_data);
```

```
else
    [data, y] = m_ary(2,K,Ns);
end

% pulse length 2*M*Ns
M = 4;
n = -M*Ns:M*Ns;
p = exp(-2*pi^2*BT^2/log(2)*(n/Ns).^2);
p = p/sum(p);

% Convert to +1/-1 data bits
data = 2*data-1;
if MSK == 0
data1 = data;
else
data1 = filter(p,1,data);
end

data = data(1:Ns:end)';
% convert back to 0/+1 data bits
data = (data+1)/2;
x = exp(j*pi/2*cumsum(data1)/Ns).';

%///////////////////////////////////////////////////////////////////
function trellis_plot(x,Ns)
%///////////////////////////////////////////////////////////////////
% trellis_plot(x,Ns)
%
%
% Mark Wickert June 2007

% Trim to remove filter delay
x = x(4*Ns:end);
hold off
K = length(x)/Ns;
M = 5;
NK = fix(K/M);
phase = unwrap(angle(x));
t = [0:M*Ns-1]/Ns;
flag = 0;
for i=0:NK-1
    if i < NK-1
        phase1 = phase(1+i*M*Ns:(i+1)*M*Ns+1);
    else
        phase1 = phase(1+i*M*Ns:(i+1)*M*Ns);
    end
```

```
    t = [1:length(phase1)]/Ns;
    phase2 = (phase1-round(phase1(1)/(pi/2))*pi/2)/(pi/2);
    if abs(phase2(1)) <= 1/20
        plot(t-1/Ns,phase2)
        if flag == 0
            hold on
        end
        flag = 1;
    end
end
axis([0 M -M M])
xlabel('Bit Intervals')
ylabel('Phase Deviation Normalized by pi/2')
grid
hold off
```

- First we compare the phase trellis between MSK and GMSK with $BT_b = 0.35$ (GSM value)

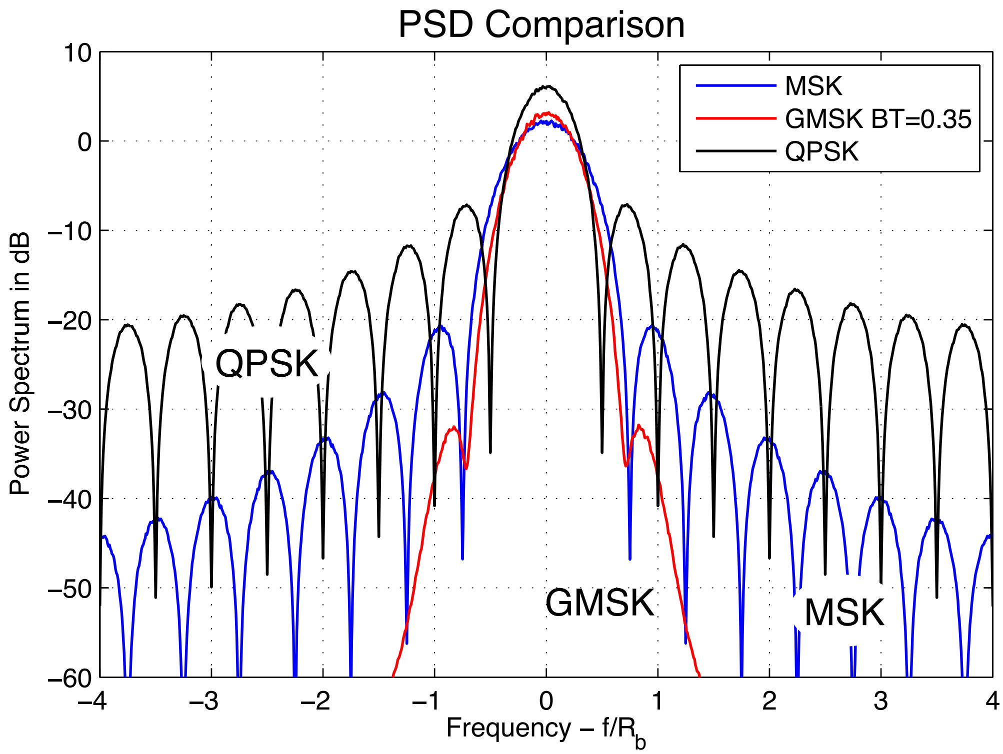
```
>> xMSK = gmsk(0.35, 2000, 16, 0);
>> xGMSK = gmsk(0.35, 2000, 16, 1);
>> subplot(211)
>> trellis_plot(xMSK,16)
>> subplot(212)
>> trellis_plot(xGMSK,16)
```



Trellis plots: MSK (top) and GMSK $BT_b = 0.35$ (bottom)

- Next we compare the PSD of MSK and GMSK with $BT_b = 0.35$ (GSM value)

```
>> xMSK = gmsk(0.35, 100000, 16, 0); % 16 samples per bit
>> xGMSK = gmsk(0.35, 100000, 16, 1); % 16 samples per bit
>> xQPSK = psk_tx('qpsk',32,100000); % 32 samples per symb/16 per bit
>> spec_plot(xMSK,2^11,16);
>> axis([-4 4 -60 10])
>> hold
Current plot held
>> spec_plot(xGMSK,2^11,16,'r');
>> spec_plot(xQPSK,2^11,16,'k');
>> legend('MSK','GMSK BT=0.35','QPSK')
```



PSD comparison MSK and GMSK $BT_b = 0.35$

6.2.4 M-ary Data Transmission in Terms of Signal Space

- The *signal space* interpretation of digital modulation, especially for M-ary schemes, is very useful for understanding performance analysis and receiver design
- In the Z&T text the first notion of signal spaces was when generalized Fourier series was developed in Chapter 2
- The context we have now is *statistical decision theory*, which is formally dealt with in Z&T Chapter 11
- At present we will use a heuristic approach

- Consider a coherent communications system where each transmitted waveform (symbol), $s_i(t)$, can be represented as

$$s_i(t) = \sum_{j=1}^{K} a_{ij}\phi_j(t),\ 0 \le t \le T_s,\ K \le M, i = 1, 2, \ldots, M$$

- The functions $\phi_j(t)$ are *orthonormal* over the symbol interval, which means that for any m, n pair

$$\langle \phi_m(t), \phi_n(t) \rangle = \int_0^{T_s} \phi_m(t)\phi_n(t)\, dt = \begin{cases} 1, & m = n \\ 0, & m \ne n \end{cases}$$

Note: For complex functions $\phi_m(t)\phi_n(t) \to \phi_m(t)\phi_n^*(t)$

- It turns out that (Z&T Chap. 11) the a_{ij}'s can be found via

$$a_{ij} = \langle s_i(t), \phi_j(t) \rangle = \int_0^{T_s} s_i(t)\phi_j(t)\, dt$$

- From the signal representation using the ϕ_i's, we can visualize each signal as a point in space with the coordinate axis being the basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_K(t)$
  - The weights a_{ij} describe the *signal space coordinates* of a particular $s_i(t)$
  - The *signal constellation* points are the vectors

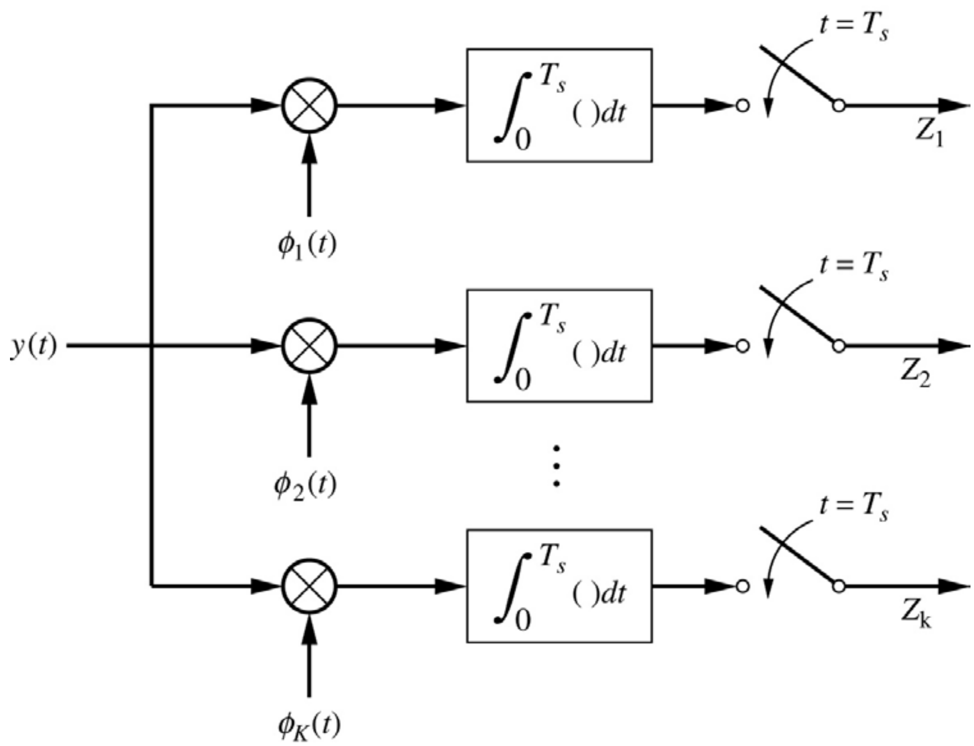
$$\mathbf{a}_i = \begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{iK} \end{bmatrix}$$

- If we include AWG channel noise $n(t)$, we can model the received signal in terms of signal space coordinates as follows

$$y(t) = s_i(t) + n(t),\ t_0 \le t \le t_0 + T_s,\ K \le M, i = 1, 2, \ldots, M$$

where t_0 is arbitrary allow for signal arrival delay at the receiver

- The signal space coordinates can be obtained using a correlator bank



Correlator bank for obtaining the Z_j's

- Each correlator has output

$$Z_j = a_{ij} + N_j, \quad j = 1, 2, \ldots, K, \quad i = 1, 2, \ldots, M$$

where the N_j are given by

$$N_j = \langle n(t), \phi_j(t) \rangle = \int_0^{T_s} n(t)\phi_j(t)\, dt$$

- The a_{ij} are deterministic, since they correspond to the signal set design

- The N_j are Gaussian random variables, since they result from *projecting* $n(t)$ onto the coordinate function $\phi_j(t)$ via the action of the correlator

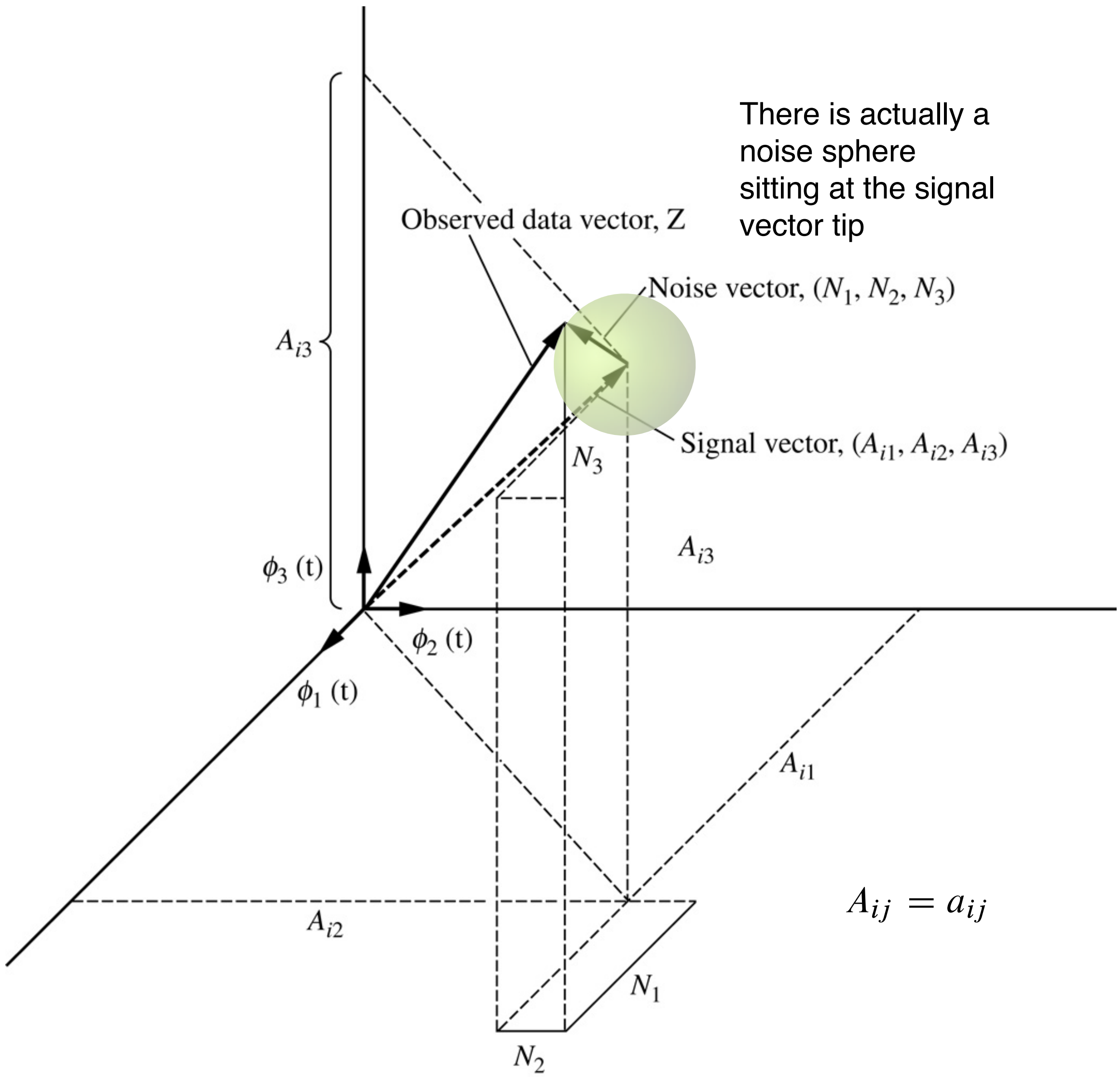
- We know that the N_j are zero mean, so to characterize them we need only find $E[N_j N_k]$

- It can be shown that with $n(t)$ having the usual $N_0/2$ two-sided PSD,

$$\begin{aligned}
E\big[N_j N_k\big] &= E\left[\int_0^{T_s} n(t)\phi_j(t)\,dt \int_0^{T_s} n(\lambda)\phi_k(\lambda)\,d\lambda\right] \\
&= \int_0^{T_s}\int_0^{T_s} \underbrace{E[n(t)n(\lambda)]}_{R_{nn}(t-\lambda)} \phi_j(t)\phi_k(\lambda)\,d\lambda dt \\
&= \frac{N_0}{2}\int_0^{T_s} \phi_j(t)\phi_k(t)\,dt \\
&= \begin{cases} N_0/2, & j = k \\ 0, & j \neq k \end{cases}
\end{aligned}$$

- It can be shown that the signal space representation preserves the information needed to make decisions on what signal was transmitted

- The signal space representation for a 3-correlator expansion is shown below



Signal model showing the Z_j's composed of signal and noise RVs

- Following the correlator bank, the receiver makes decisions based on the reference vector whose a_{ij} minimize the squared Euclidean distance

$$d_i^2 = \sum_{j=1}^{K} \left[Z_j - a_{ij} \right]^2$$

Note: This rule gives the minimum probability of error receiver (for equally likely signals)

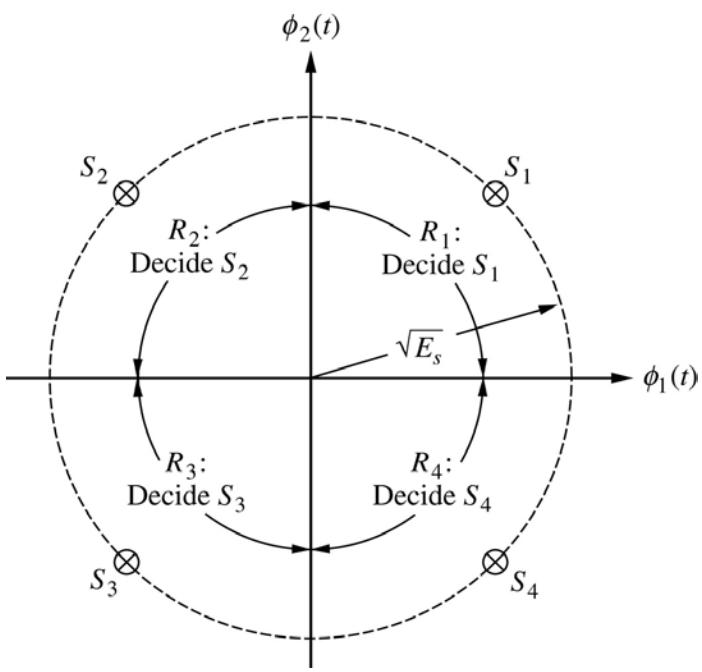
Example 6.2: **QPSK in Signal Space**

- For QPSK we need two basis functions, since we know that $x_c(t)$ was originally written in terms of sin() and cos()
- With basis functions

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t), \quad 0 \le t \le T_s$$
$$\phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t), \quad 0 \le t \le T_s$$

- We now can write that

$$\begin{aligned} x_c(t) = s_i(t) &= \sqrt{E_s}\big[d_1(t)/\sqrt{2}\cdot\phi_1(t) - d_2(t)/\sqrt{2}\cdot\phi_2(t)\big] \\ &= \sqrt{E_s}\big[\pm\phi_1(t) \pm \phi_2(t)\big]/\sqrt{2}\end{aligned}$$



QPSK signal space including the decision boundaries

- Note that the decision boundaries follow from the minimum distance decision rule, here they are just the quadrants of the plane

- The signal point denoted $S_1 = (\sqrt{E_s/2}, \sqrt{E_s/2}) = (\sqrt{E_b}, \sqrt{E_b})$ corresponds to $(d_1(t), d_2(t)) = (1, 1)$; Recall $E_s = 2E_b$

- When noise is present the signal vector is moved away from S_1 by the noise vector having components (N_1, N_2)

- Due to symmetry the conditional error probabilities, i.e., the probability of lying outside the intended quadrant, are all equal

- The probability of making an error is thus

$$\begin{aligned} P_E &= P\big[Z \in R_2 \text{ or } R_3 \text{ or } R_4 | S_1 \text{ sent}\big] \\ &< P\big[Z \in R_2 \text{ or } R_3 \,|S_1 \text{ sent}\big] + P\big[Z \in R_3 \text{ or } R_4 \,|S_1 \text{ sent}\big] \end{aligned}$$

- The over bounding of P_E comes by double counting the probability of landing in region R_3, but the probabilities themselves involve only a single random variable; in fact both terms are identical

- We simply need to find the probability that the received correlator output Z, corresponding to S_1, has either a negative $\phi_1(t)$ component or a negative $\phi_2(t)$ component

- Considering that the $\phi_1(t)$ component is negative we have

$$\begin{aligned} P_E &< 2P\big[Z \in R_2 \text{ or } R_3\big] = 2P\big[\sqrt{E_s/2} + N_\perp < 0\big] \\ &= 2P\left[N_\perp < -\sqrt{\frac{E_s}{2}}\right] \end{aligned}$$

where $N_\perp$ is the noise $n(t)$ projected along $\phi_1(t)$

- We know that $N_\perp$ is zero mean and has variance $N_0/2$, so

$$P_E < 2\int_{-\infty}^{-\sqrt{E_s/2}} \frac{e^{-u^2/N_0}}{\sqrt{\pi N_0}}\, du = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

The same bound as obtained earlier!

6.2.5 M-ary Phase Shift Keying

- We now expand the QPSK definition to M phases

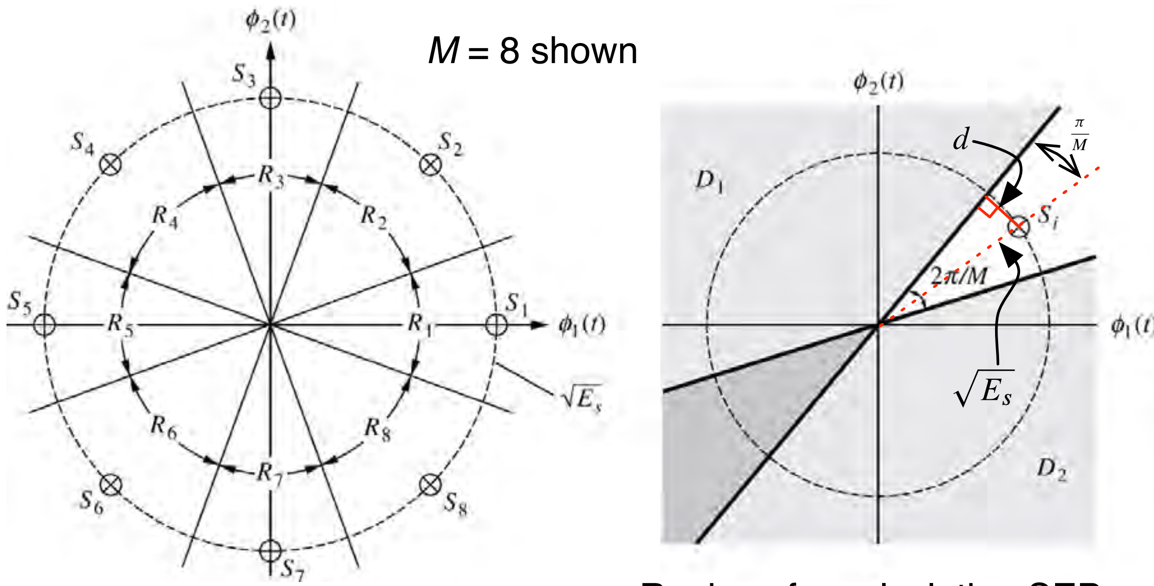
$$s_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t + \frac{2\pi(i-1)}{M}\right), \\ 0 \le t \le T_s,\ i = 1, 2, \ldots, M$$

- The basis functions are identical to QPSK, so we can expand $s_i(t)$ as

$$\begin{aligned} s_i(t) = \sqrt{E_s}\Bigg[&\cos\left(\frac{2\pi(i-1)}{M}\right)\phi_1(t) \\ &- \sin\left(\frac{2\pi(i-1)}{M}\right)\phi_2(t)\Bigg], \end{aligned}$$

where $\phi_1(t)$ and $\phi_2(t)$ are as defined in the QPSK signal space example

- The signal space for M-ary PSK is shown below



Signal space

Regions for calculating SEP and its bound

M-ary PSK signal space and regions for calculating P_E

- The decision boundaries are formed by the angle value that is midway between the signal points themselves, e.g.,

$$\text{decision boundaries} = \arg\left[S_i\right] \pm \frac{\pi}{M}$$

- Symbol errors occur when the received signal vector lies outside the intended region

- A simple bound and an exact calculation of P_E is possible

- To bound P_E consider the overlapping regions D_1 and D_2 in the above figure

$$P_E < P\left[Z \in D_1 \text{ or } D_2\right] = 2P\left[Z \in D_1\right]$$

- The shortest distance from the signal point S_i to the decision boundary is given by

$$d = \sqrt{E_s}\sin\left(\frac{\pi}{M}\right)$$

- Noise at S_i is circularly symmetric, so the variance in any direction is $N_0/2$, meaning that the probability of the noise exceeding d is the probability of interest, i.e.,

$$P_E < 2P\big[N_\perp > d\big] = 2\int_d^\infty \frac{e^{u^2/N_0}}{\sqrt{\pi N_0}}\,du$$
$$= 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

- An exact expesssion for P_E is[1]

$$\boxed{P_E = \frac{1}{\pi}\int_0^{\pi-\pi/M} \exp\left(-\frac{(E_s/N_0)\sin^2(\pi/M)}{\sin^2\phi}\right)d\phi}$$

See Z&T pp. 494–495 for details on the derivation

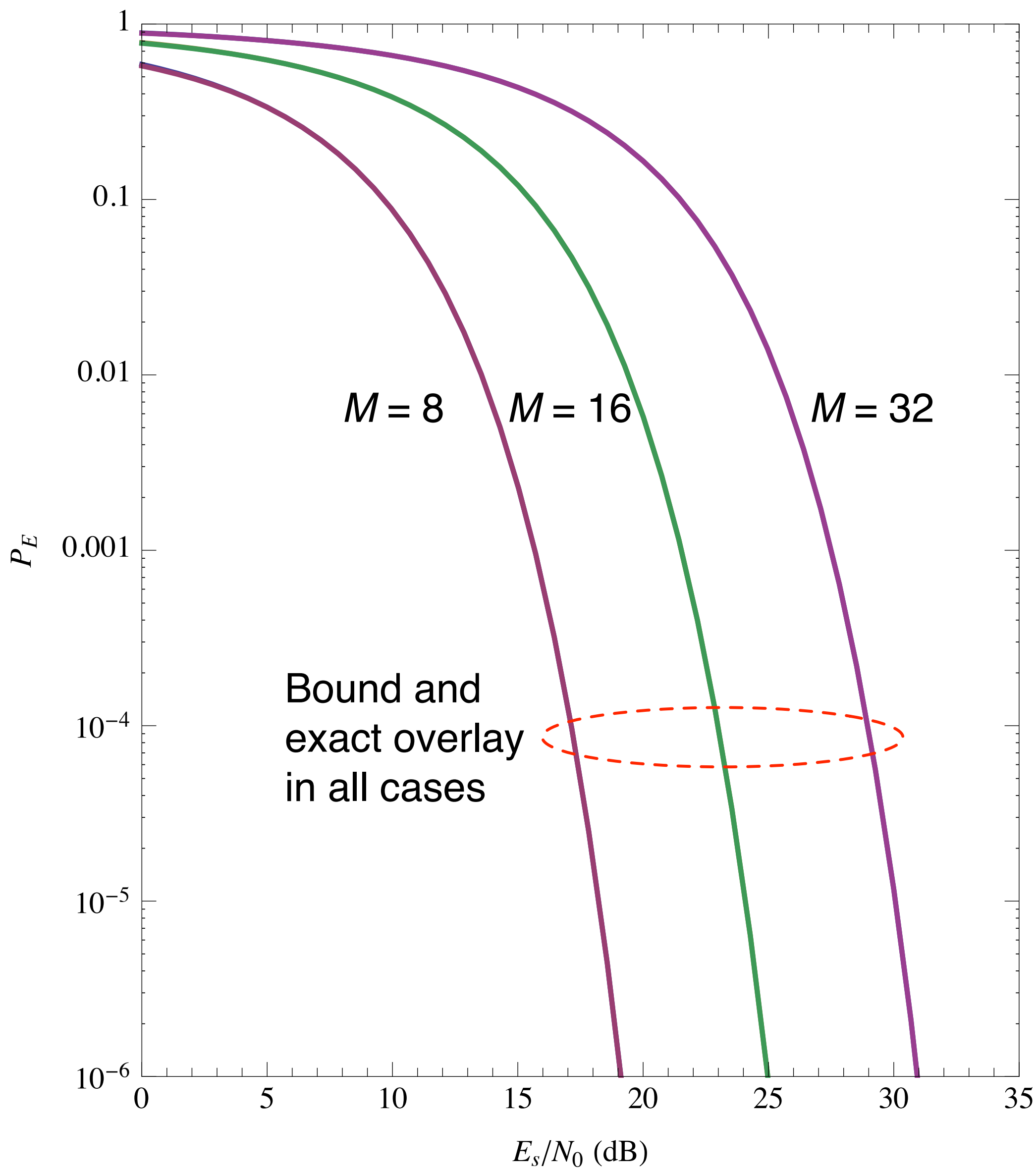
Example 6.3: **MPSK P_E Formula Compare**

- In this example we compare the approximate *union bound* formula for P_E with the exact formula

- We consider $M = 8$, 16, and 32 using Mathematica for the numerical integration

[1] J.W. Craig, "A New, Simple andExact Result for Calculating the Probability of Error for A Two-Dimensional Signal Constellations," *IEEE Milcom '91 Proceedings*, pp. 571–575, October 1991.

```
PEpsk[zdB_, M_] :=
  1/π NIntegrate[Exp[-10^(z/10) (Sin[π / M])^2 / (Sin[φ])^2] /. {M → 8, z → 5}, {φ, 0, π - π / M}];
LogPlot[{2 Q[Sqrt[2 × 10^(z/10)] Sin[π / 8]], PEpsk[z, 8],
  2 Q[Sqrt[2 × 10^(z/10)] Sin[π / 16]], PEpsk[z, 16], 2 Q[Sqrt[2 × 10^(z/10)] Sin[π / 32]], PEpsk[z, 32]},
 {z, 0, 35}, PlotStyle → AbsoluteThickness[1.5], AspectRatio → 1.2,
 FrameLabel → {"Es/N0 (dB)", "PE"}, Frame → True, PlotRange → {{0, 35}, {10^-6, 1}}]
```



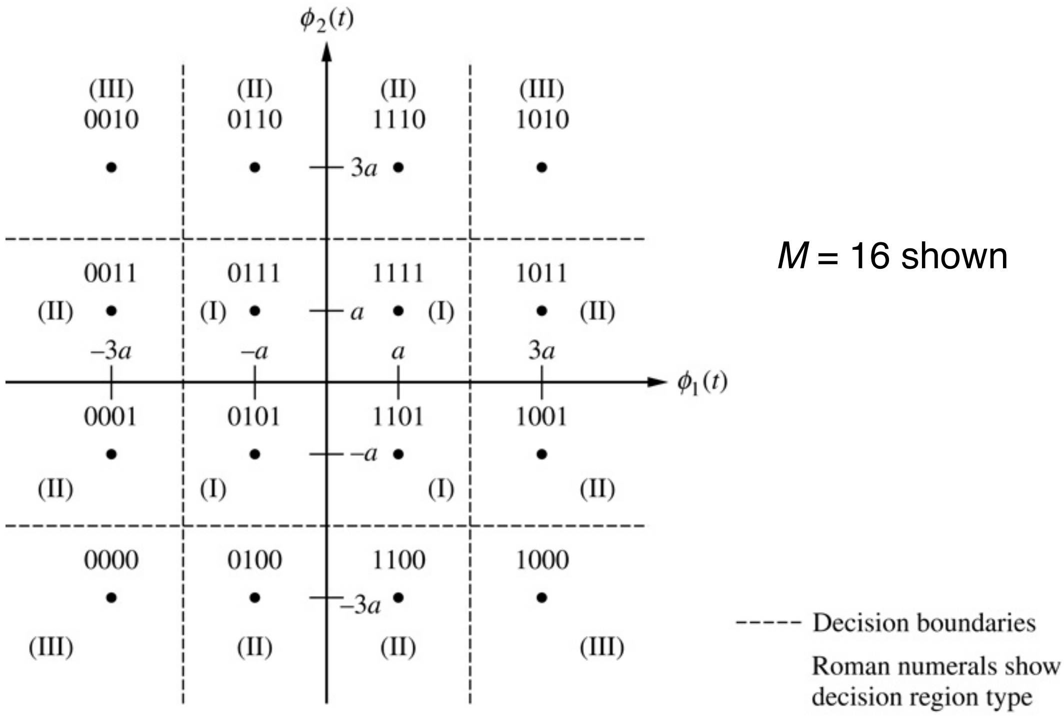
MPSK SEP bound and exact compare, $M = 8$, 16, 32

6.2.6 Quadrature-Amplitude Modulation (QAM)

- With QAM we allow amplitude values A_i and B_i of the form $\pm a, \pm 3a, \ldots, \pm(\sqrt{M}-1)a$ in both the I and Q signal coordinates, thus

$$s_i(t) = \sqrt{\frac{2}{T_s}}\big[A_i \cos(2\pi f_c t) + B_i \sin(2\pi f_c t)\big], \quad \begin{array}{l} 0 \leq t \leq T_s \\ i = 1, \ldots, M \end{array}$$

- Again there are just two basis functions, $\phi_1(t)$ and $\phi_2(t)$

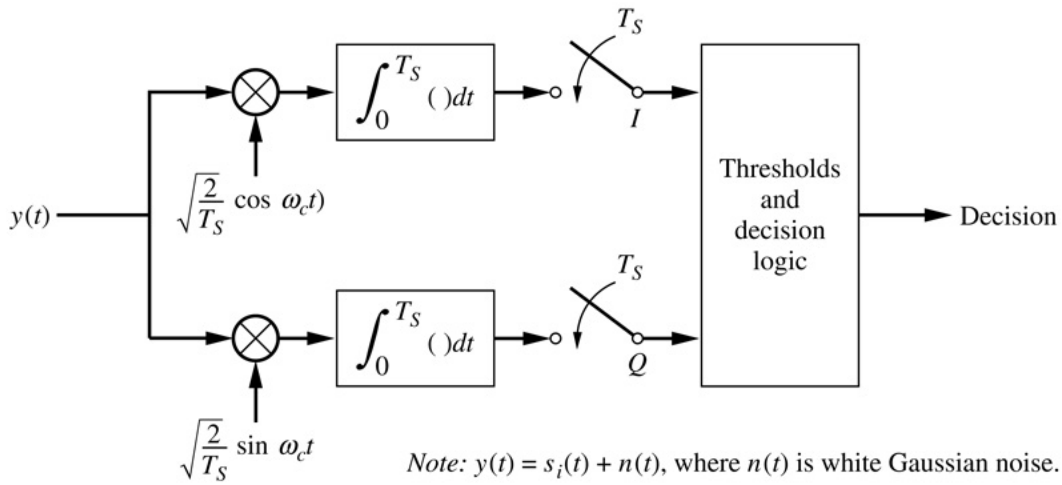


The QAM signal space

- The average signal energy E_s is related to a via

$$a = \sqrt{\frac{3E_s}{2(M-1)}}$$

- The optimal receiver again employs just two correlators



QAM receiver

- SEP calculation is tedious, as there are three (I, II, & III) different regions to consider
- When all is accounted for

$$P_E = 1 - \frac{1}{M}\big[(\sqrt{M}-2)^2 P(C|\text{I}) + 4(\sqrt{M}-2)P(C|\text{II}) + 4P(C|\text{III})\big]$$

where C means the probability of being correct, and the I's are the conditioning region/symbol types

- At the correlator output we know that the noise variance along each component, ϕ_1 and ϕ_2, is $\sigma_N^2 = N_0/2$
- For the type I symbol, the probability of lying inside the square region along the ϕ_1 axis is $1-2Q\big(\sqrt{2a^2/N_0}\big)$ and similarly for the ϕ_2 axis, since these two events are independent the probability of being correct is

$$P(C|\text{I}) = \left[1 - 2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

- For the type II region there is only one boundary on either the ϕ_1 or ϕ_2 axes, and two along the other, so we have a mixed case in finding the probability of being correct

$$P(C|\text{II}) = \left[1 - 2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]\left[1 - Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]$$

- The type III symbol lies at the corner, so when committing an error, there is only one decision boundary to cross in the ϕ_1 and ϕ_2 directions, thus

$$P(C|\text{III}) = \left[1 - Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

- The remaining factors are the number of each region type, i.e., 4 corners (tpe III), $4(\sqrt{M}-2)$ sides (type II), and $\left(\sqrt{M}-2\right)^2$ interior regions (type I)

- To work with this formula it is best to write a function in either MATLAB or Mathematica

- It is worth noting that any term that involves $Q^2(\,)$ can be ignored so long as $E_s/N_0 \gg 1$, thus a simplifying approximation is

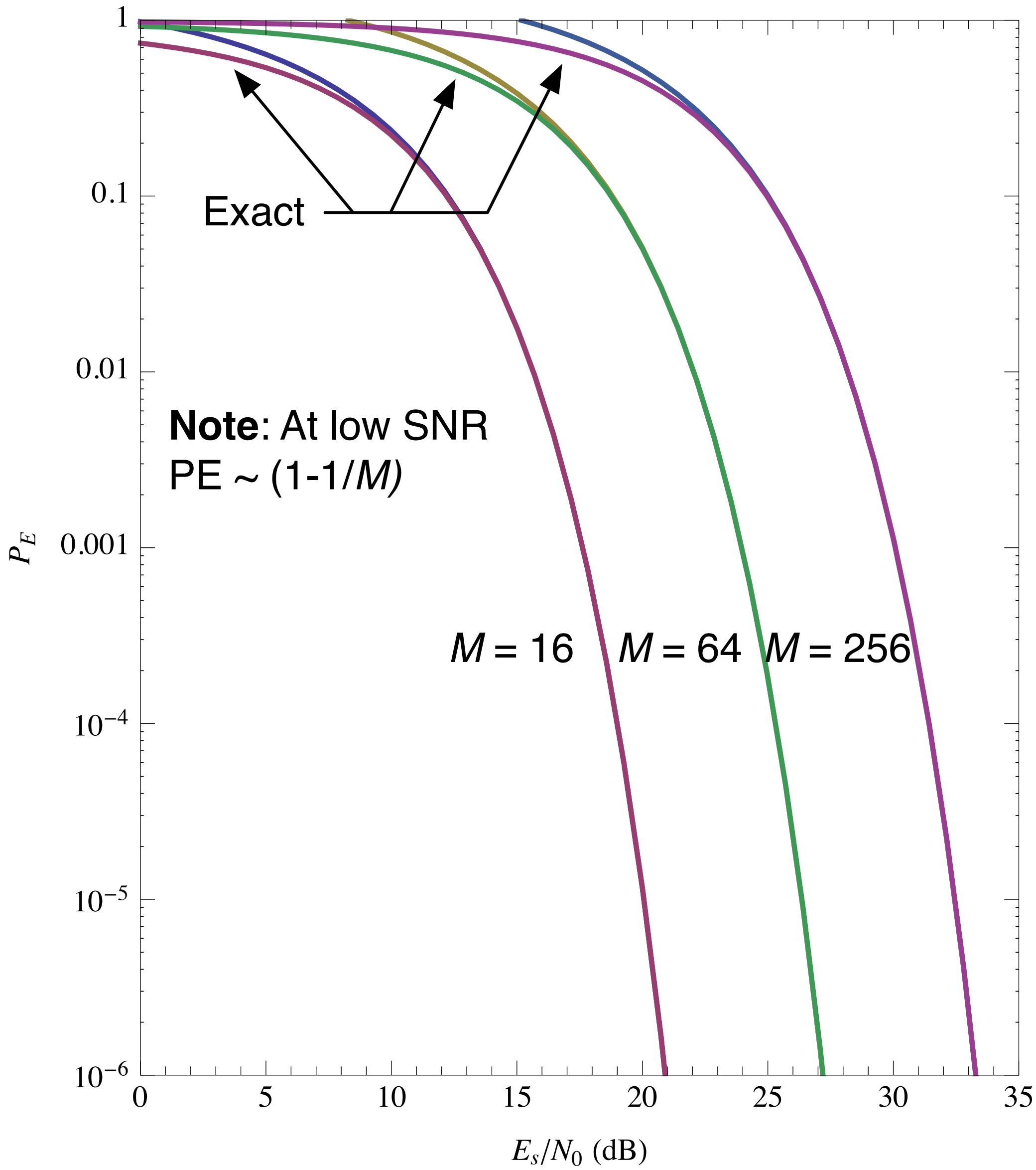
$$P_E \simeq 4\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right),\ \frac{E_s}{N_0} \gg 1$$

Example 6.4: M-QAM P_E Formula Compare

- In this example we compare the approximate formula for P_E with the exact formula
- We consider $M = 8$, 16, and 32 using Mathematica

```
PEqam[zdB_, M_] := Module[{a, Pc1, Pc2, Pc3},
  a = Sqrt[(3 × 10^(zdB/10))/(2 (M - 1))];
  Pc1 = (1 - 2 Q[Sqrt[2 a^2]])^2;
  Pc2 = (1 - 2 Q[Sqrt[2 a^2]]) (1 - Q[Sqrt[2 a^2]]);
  Pc3 = (1 - Q[Sqrt[2 a^2]]) (1 - Q[Sqrt[2 a^2]]);
  1 - 1/M ((Sqrt[M] - 2)^2 Pc1 + 4 (Sqrt[M] - 2) Pc2 + 4 Pc3)
];
```

Mathematica function for P_E exact



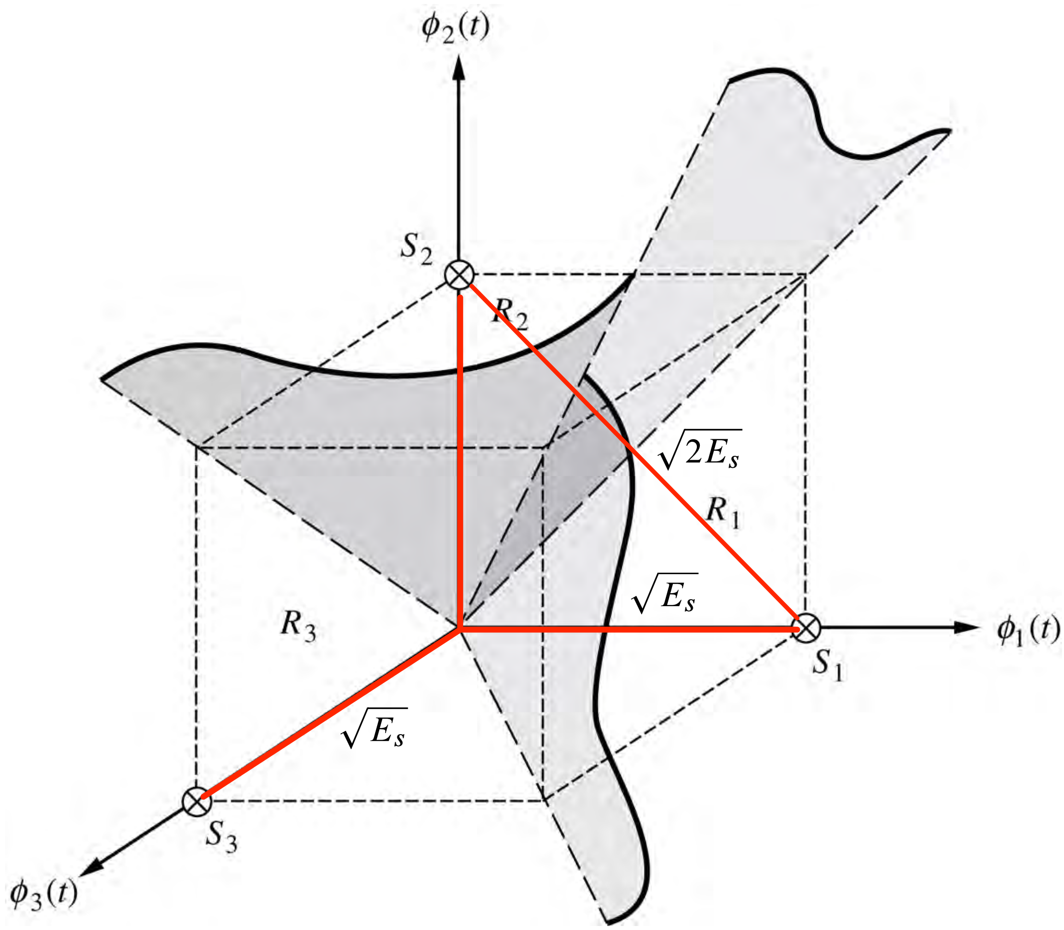
M-QAM SEP bound and exact compare, $M = 8$, 16, 64

6.2.7 Coherent FSK

- M-ary FSK is an extension of binary FSK from notes Chapter 5 (Z&T Chapter 9)

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left\{2\pi\left[f_c + (i-1)\Delta f\right]t\right\},\ 0 \le t \le T_s,$$
$$i = 1, 2, \ldots, M$$

- For $\Delta f = 1/(2T_s)$ the signals are mutually orthogonal, the same holds as $\Delta f\, T_s$ becomes large



M-ary Coherent FSK signal space $M = 3$ (easy to draw)

- The signal space is thus M-dimensional with orthonormal basis functions of the form

$$\phi_i(t) = \sqrt{\frac{2}{T_s}} \cos\left\{2\pi\left[f_c + (i-1)\Delta f\right]t\right\},\ 0 \le t \le T_s,$$
$$i = 1, 2, \ldots, M$$

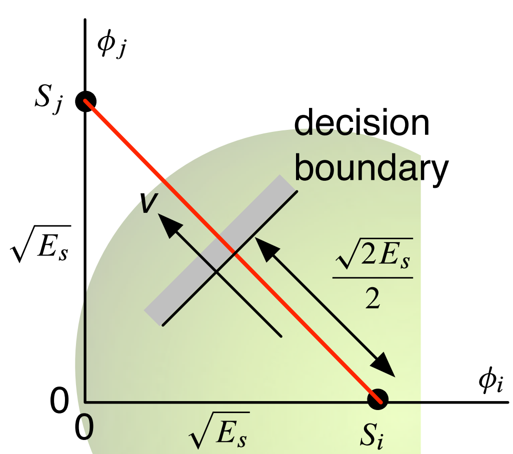
- The ith signal, in terms of the basis functions, is

$$s_i(t) = \sqrt{E_s}\phi_i(t)$$

- The symbol error probability can be obtained via the union bound

- Symbol S_i is in error if it crosses into the S_j decision region for $j \ne i$

- Considering any pair of dimensions we have the following decision condition



P_E calculation

$$\begin{aligned} P(A_{ij}) &= P(S_i \text{ in } S_j \text{ region}|S_i \text{ sent}) \\ &= \int_{\sqrt{2E_s}/2}^{\infty} \frac{1}{\sqrt{2\pi(N_0/2)}} e^{v^2/(2N_0/2)}\, dv \\ &= Q\left(\sqrt{\frac{E_s}{N_0}}\right) \end{aligned}$$

- The error probability is the union of all A_{ij} events

$$\boxed{\begin{aligned} P_E &= P\left(\bigcup_{\substack{j=1 \\ j\neq i}}^{M} A_{ij}\right) \leq \sum_{\substack{j=1 \\ j\neq i}}^{M} P\big(A_{ij}\big) \\ &= (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right) \end{aligned}}$$

Example 6.5: M-ary FSK in MATLAB

- The function `fsk.m` written for binary FSK can be modified to produce MFSK

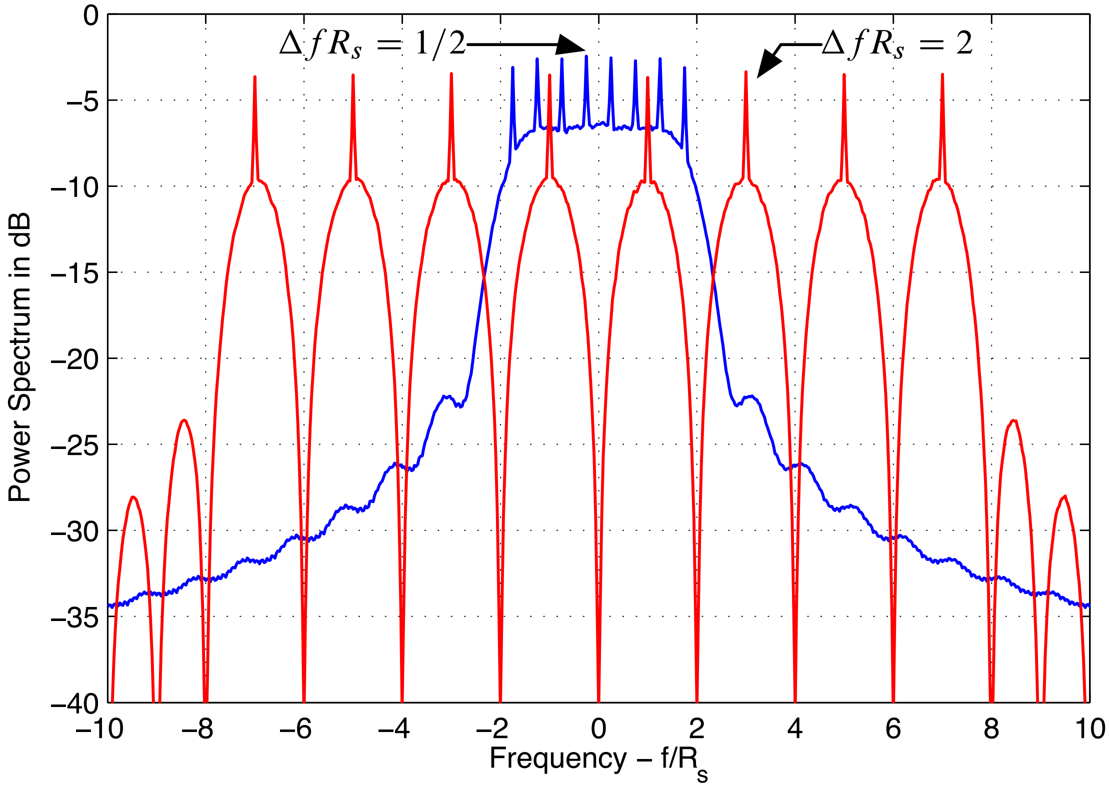
```
function [x,data,x_ref] = mfsk(M,Df, Nbit, Ns)
%        [x,data,x_ref] = mfsk(M,Df, Nbit, Ns)
%
% Complex baseband M-ary FSK modulator
%///////////////////////////////////////////////////////////////////////
%      M = the number of levels (typically a power of two)
%     Df = peak-to-peak frequency deviation normalized to bit rate
%   Nbit = number of symbols processed
%  Ns   = the number of samples per bit
%
%      x = Complex baseband transmit signal vector
%   data = Binary 0,1 data being sent; used for error detecting
%  x_ref = Complex baseband reference signal used for coherent
%          demodulation of FSK (does not contain data)
```

```
%/////////////////////////////////////////////////////////////////////
% Mark Wickert November 2010

A = 1;
data = randi([0,M-1],1,Nbit); % 0,1 random integers
y = [data-(M-1)/2; zeros(Ns-1,Nbit)]; %impulse train modulate +/-1
y = reshape(y,1,Nbit*Ns);
y = filter(ones(1,Ns),1,y); % apply REC pulse shape

n = (0:length(y)-1); % time axis for generating FM

x = A*exp(1i*2*pi*Df*y.*n/Ns);
% reference for coherent correlation at receiver
x_ref = A*exp(1i*2*pi*Df/2.*n/Ns);
```

- Consider $M = 8$ under with $\Delta f R_s = 1/2$ and 2

```
>> [x05,data,x_ref] = mfsk(8,1/2, 100000, 32);
>> [x20,data,x_ref] = mfsk(8,2, 100000, 32);
>> spec_plot(x05,2^10,32);
>> hold
Current plot held
>> spec_plot(x20,2^10,32,'r');
>> grid
>> xlabel('Frequency - f/R_s')
>> axis([-10 10 -40 0])
```

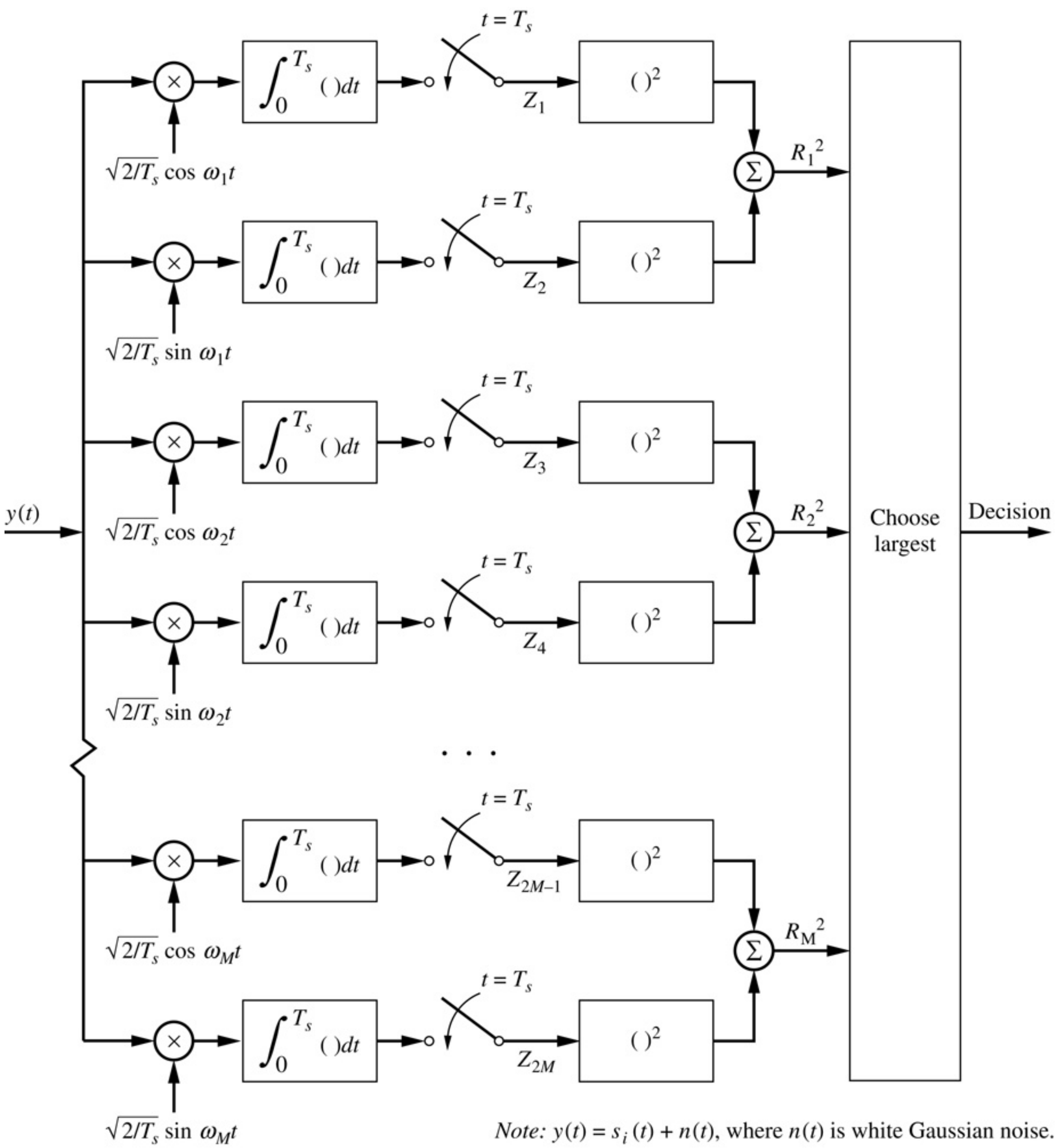
- Notice that the parameter `Ns` has to be increased to give adequate spectral space for large `M` and `Df`



PSD of MFSK when $M = 8$ and $\Delta R_s = 1/2$ and 2

6.2.8 Noncoherent FSK

- Noncoherent M-ary FSK (MFSK) uses the same transmitted signal as coherent FSK
- The receiver can be a bank of bandpass filters and envelope detectors



Noncoherent MFSK receiver

- The symbol error probability can be obtained through a rather lengthly calculation

- In Z&T a proof sketch is given, with the result being

$$P_E = \sum_{k=1}^{M-1} \binom{M-1}{k} \frac{(-1)^{k+1}}{k+1} \exp\left(-\frac{k}{k+1} \cdot \frac{E_s}{N_0}\right)$$

6.2.9 Differentially Coherent PSK (DPSK)

- Binary DPSK was introduced in notes Chapter 5 (Z&T Chapter 9)

- Extending from the binary case, MDPSK, chooses successive transmitted signals to be differential phase relative to the present signaling interval, i.e.,

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t),\ 0 \le t \le T_s$$
$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi(i-1}{M}\right),\ T_s \le t \le 2T_s$$

  where $i = 2, \ldots M$

- Assuming that the channel phase is constant over a two symbol interval, or more, the present symbol can be used to demodulate the following symbol

- With AWGN present the receiver must discern which $2\pi/M$ phase step was taken on the $[T_s, 2T_s]$ interval

- It can be shown that (see references in Z&T p. 503) that

$$P_E = \frac{1}{\pi}\int_0^{\pi-\pi/M} \exp\left(-\frac{(E_s/N_0)\sin^2(\pi/M)}{1+\cos(\pi/M)\cos\phi}\right) d\phi$$

6.2.10 BEP from SEP

- When comparing modulation schemes it is best to use BEP
- When symbols are *Gray* encoded, that is adjacent symbols in the symbol space differ by only one bit, then we can use the SEP expressions to lower bound the BEP
  - More information on Gray encoding can be found in Z&T problem 9.32
- Assuming that $P_{E,\text{symb}}$ is small, symbol errors will be dominated by single bit errors, so we can write that

$$P_{E,\text{bit}} \simeq \frac{P_{E,\text{symb}}}{\log_2 M}$$

  Note this is actually a lower bound, since multiple bit errors within a symbol would raise this value

- For the case of orthogonal signaling, another approximation is to consider the natural binary assignment made in representing, say an $M = 8$ symbol

BEP from SEP for Orthogonal Signaling

M-ary Signal	Binary Representation		
1 or (0)	0	0	0
2 or (1)	0	0	1
3 or (2)	0	1	0
4 or (3)	0	1	1
5 or (4)	1	0	0
6 or (5)	1	0	1
7 or (6)	1	1	0
8 or (7)	1	1	1

- Taking the right column, for instance, we see that when a bit error occurs in this column, there are $M/2$ of a possible of $M-1$ ways that a bit may be in error, thus

$$P(B|S) = \frac{M/2}{M-1}$$

- Using Bayes' rule it follows that for orthogonal modulation (i.e., MFSK)

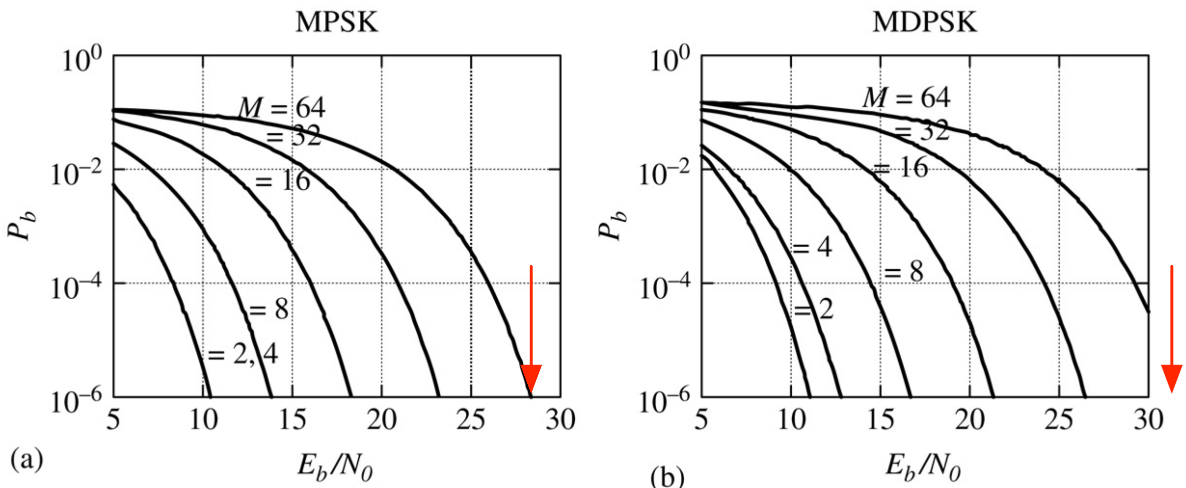
$$P_{E,\text{bit}} = \frac{M}{2(M-1)} P_{E,\text{symb}}$$

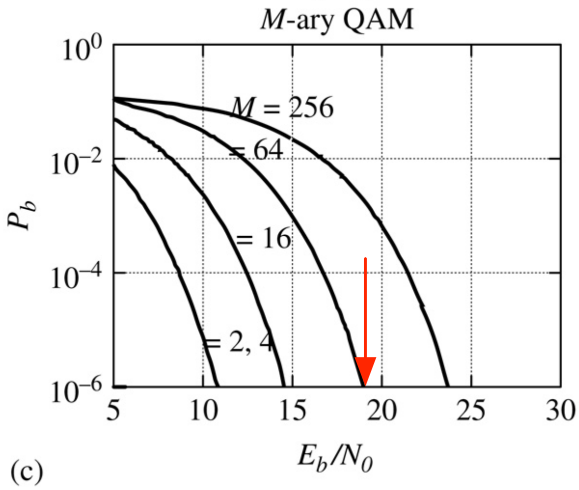
- For all M-ary schemes we also desire to compare them in terms of E_b rather than E_s, i.e.,

$$\boxed{E_s = (\log_2 M) E_b}$$

Example 6.6: *M*-ary BEP Power Efficiency Comparisons

- First we compare MPSK and M-ary QAM, and MDPSK

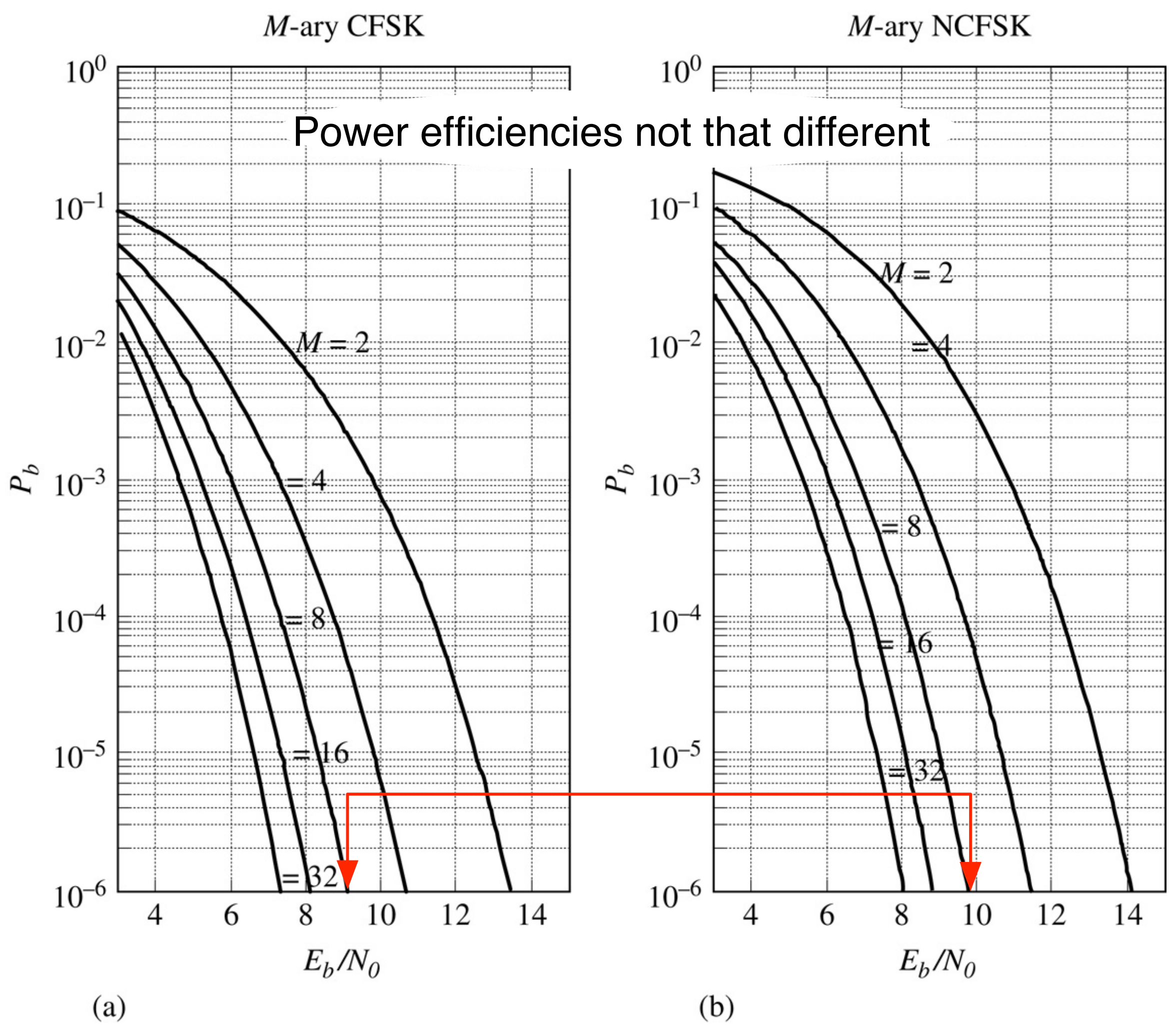




In terms of power efficiency: M-ary QAM is best, followed by MPSK and then MDPSK

P_b comparison: MQAM and MDPSK

- Next we compare M-ary CFSK and M-ary NCFSK



P_b comparison: CFSK and NCFSK

- It is not a mistake that the power efficiency in terms of BEP and energy per bit, actually increases as M increases
- We will see shortly that the bandwidth efficiency is poor

Example 6.7: M-ary BEP Bandwidth Efficiency Comparisons

- Bandwidth efficiency is also important
- If we use the approximate null-null bandwidth for PSK and QAM, assuming a rectangular pulse shape, we have

$$B_{\text{MPSK,M-QAM}} = \frac{2}{T_s} = \frac{2R_b}{\log_2 M} \text{ Hz}$$

- For coherent FSK we have he following approximate bandwidth, assuming $\Delta f = 1/(2T_s)$

$$\begin{aligned} B_{\text{CFSK}} &= \underbrace{\frac{1}{T_s}}_{\text{left end}} + \underbrace{\frac{M-1}{2T_s}}_{\text{inbetween}} + \underbrace{\frac{1}{T_s}}_{\text{right end}} \\ &= \frac{M+3}{2T_s} = \frac{(M+3)R_b}{2\log_2 M} \text{ Hz} \end{aligned}$$

- For noncoherent MFSK we increase Δf to about $2/T_s$, so

$$\begin{aligned} B_{\text{NCFSK}} &= \underbrace{\frac{1}{T_s}}_{\text{left end}} + \underbrace{\frac{2(M-1)}{2T_s}}_{\text{inbetween}} + \underbrace{\frac{1}{T_s}}_{\text{right end}} \\ &= \frac{2M}{T_s} = \frac{2MR_b}{\log_2 M} \text{ Hz} \end{aligned}$$

Bandwidth efficiency comparisons in bits/s/Hz

M	QAM	PSK	Coherent FSK	Noncoherent FSK
2		0.5	0.4	0.25
4	1	1	0.57	0.25
8		1.5	0.55	0.19
16	2	2	0.42	0.13
32		2.5	0.29	0.08
64	3	3	0.18	0.05

6.3 Power Spectra for Quadrature Modulation Techniques

- The previous example dealt with bandwidth efficiency, and in so doing required approximate power spectral density estimates

- In many cases the exact power spectrum of a digital modulation scheme is needed to perform some analysis

- One case in point is the calculation of fractional out-of-band power

- To establish this definition we first define the fraction of the total signal power in bandwidth B centered on the carrier (centered about zero for a complex baseband representation)

$$\Delta P_{\text{IB}} = \frac{2}{P_T} \int_{f_c - B/2}^{f_c + B/2} S(f)\, df$$

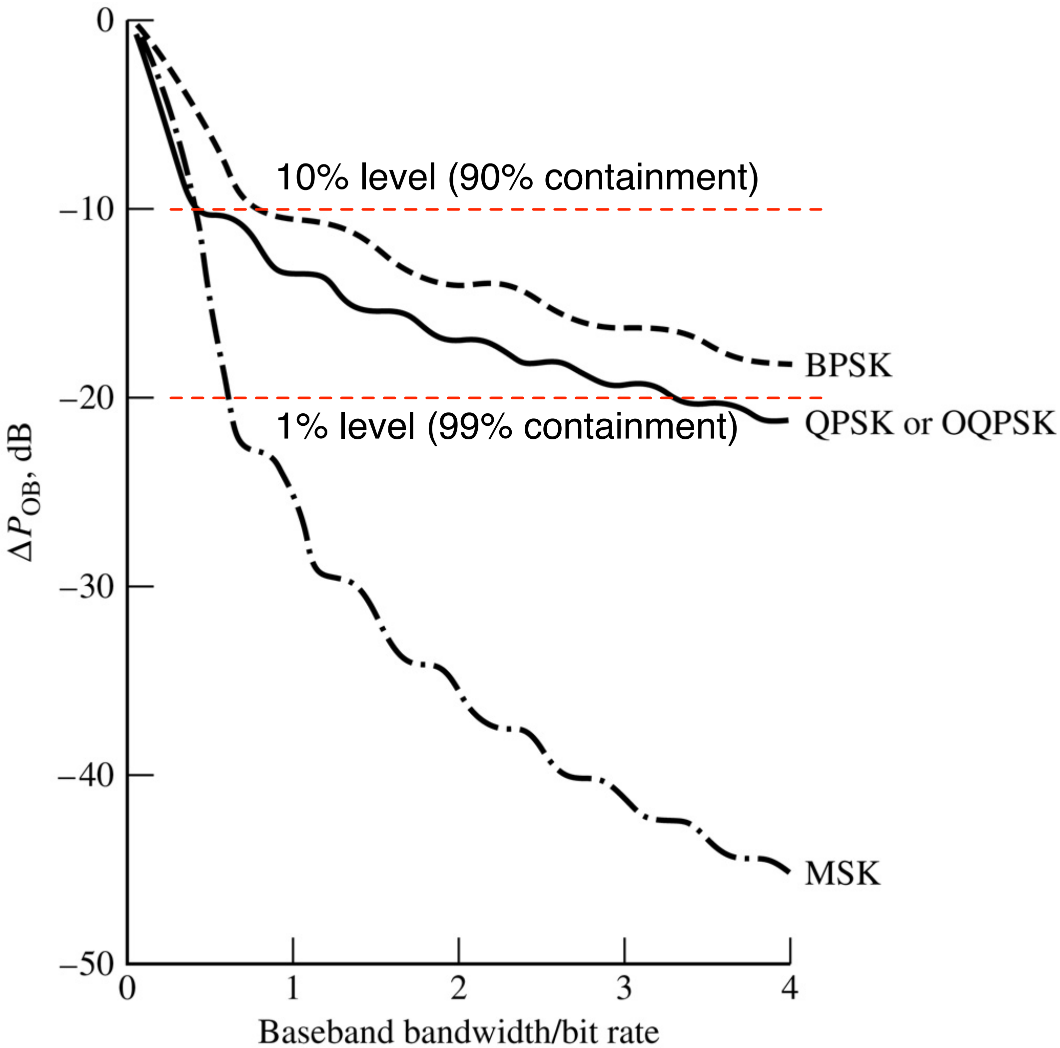
where $S(f)$ is the signal PSD and P_T is the total signal power, which can be obtained from

$$P_T = \int_{-\infty}^{\infty} S(f)\, df$$

- The percent out-of-band power ΔP_{OB} is defined as

$$\boxed{\Delta P_{\text{OB}} = (1 - \Delta P_{\text{IB}}) \times 100\%}$$

- It is customary to plot ΔP_{OB} in dB, that is $10\log_{10}(\Delta P_{\text{OB}}\text{ fraction})$ versus B or the baseband bandwidth $B/2$



ΔP_{OB} in dB versus the baseband bandwidth $(B/2)/R_b$

6.3.1 Power Spectra Calculation

- In the random process chapter (notes Chapter 4, Z&T Chapter 7), we found the power spectrum of a random binary pulse train

- With the quadrature schemes we have been discussing, we have two such sequences $d_1(t)$ and $d_1(t)$

$$d_1(t) = \sum_{k=\infty}^{\infty} a_k p(t - kT_s - \Delta_1)$$
$$d_2(t) = \sum_{k=\infty}^{\infty} b_k q(t - kT_s - \Delta_2)$$

where $p(t)$ and $q(t)$are the inphase and quadrature pulse shapes, and Δ_1 and Δ_2 are time-of-arrival randomizing rv which insure that $d_1(t)$ and $d_2(t)$ are wide sense stationary (they are already cyclostationary)

- For our model, we further assume that $\{a_k\}$ and $\{b_k\}$ are independent and identically distributed (iid) with

$$E\{a_k\} = E\{b_k\} = 0, \quad E\{a_k a_l\} = A^2\delta_{kl}, \quad E\{b_k b_l\} = B^2\delta_{kl}$$

- In complex baseband terms, we seek the PSD of

$$\tilde{z}(t) = d_1(t) + jd_2(t)$$

in the bandpass signal

$$x_c(t) = \text{Re}\big[\tilde{z}(t)\exp(j2\pi f_c t)\big]$$

- From the random process chapter the PSD of $\tilde{z}(t)$ is

$$G(f) = \lim_{T\to\infty} \frac{E\{\left|\mathcal{F}[\tilde{z}_{2T}(t)]\right|^2\}}{2T}$$
$$= \lim_{T\to\infty} \frac{E\{|D_{1,2T}(f)|^2 + |D_{2,2T}(f)|^2\}}{2T},$$

where

$$D_{1,2T}(f) = \mathcal{F}\big[d_{1,2T}(t)\big] = \sum_{k=-N}^{N} a_k P(f) e^{-j2\pi(kT_s+\Delta_1)}\big],$$
$$D_{2,2T}(f) = \mathcal{F}\big[d_{2,2T}(t)\big] = \sum_{k=-N}^{N} b_k Q(f) e^{-j2\pi(kT_s+\Delta_2)}\big],$$

$P(f) = \mathcal{F}[p(t)]$, and $Q(f) = \mathcal{F}[q(t)]$

- Examining the $E\{|D_{1,2T}(f)|^2\}$ term we have

$$E\{|D_{1,2T}(f)|^2\} = E\left\{\sum_{k=-N}^{N} a_k P(f) e^{-j2\pi(kT_s+\Delta_1)} \sum_{l=-N}^{N} a_l P^*(f) e^{j2\pi(lT_s+\Delta_1)}\right\}$$
$$= |P(f)|^2 \sum_{k=-N}^{N}\sum_{l=-N}^{N} \underbrace{E\{a_k a_l\}}_{A^2\delta_{kl}} e^{-j2\pi(k-l)T_s}$$
$$= A^2(2N+1)|P(f)|^2$$

and similarly

$$E\{|D_{2,2T}(f)|^2\} = B^2(2N+1)|Q(f)|^2$$

- **Note**: $2T = (2N+1)T_s$, so when inserting back into the limits for $G(f)$, we have

$$G(f) = \frac{A^2|P(f)|^2 + B^2|Q(f)|^2}{T_s}$$

and

$$S_{x_c}(f) = \frac{G(f - f_c) + G(f + f_c)}{2}$$

BPSK

- Here we have $p(t) = \Pi(t/T_b)$ (unless we use a zero ISI pulse shape, which also works) and $q(t) = 0$, so

$$G_{\text{BPSK}}(f) = A^2 T_b \text{sinc}^2(T_b f)$$

QPSK and OQPSK

- Here we have $p(t) = q(t) = \Pi(t/(2T_b))$ (unless we use a zero ISI pulse shape, which also works) and $A = B$, so

$$G_{\text{QPSK}}(f) = \frac{2A^2|P(f)|^2}{2T_b} = 2A^2 T_b \text{sinc}^2(2T_b f)$$

- This result also holds for OQPSK, since the time delay on the quadrature channel is not seen in $|Q(f)|^2$

M-ary QAM

- In the QPSK analysis we let $A^2 = B^2$ be the mean-square values of the I and Q amplitudes, $T_s = (\log_2 M)T_b$, and

$$p(t) = q(t) = \frac{1}{\sqrt{\log_2 M}} \Pi\left(\frac{t}{(\log_2 M)T_b}\right),$$

then we have

$$G_{\text{MQAM}}(f) = \frac{2A^2|P(f)|^2}{(\log_2 M)T_b} = 2A^2 T_b \text{sinc}^2\big[(\log_2 M)T_b f\big]$$

MSK

- Similar to QPSK or OQPSK, we just need to consider the correct pulse shape, which here is

$$p(t) = q(t - T_b) = \cos\left(\frac{\pi t}{2T_b}\right) \Pi\left(\frac{t}{2T_b}\right)$$

- The Fourier transform of this pulse (Z&T pbm 9.25) is

$$P(f) = \mathcal{F}\{p(t)\} = \frac{4T_b \cos(2\pi T_b f)}{\pi\big[1 - (4T_b f)^2\big]}$$

- Finally,

$$G_{\text{MSK}}(f) = \frac{16A^2 T_b \cos^2(2\pi T_b f)}{\pi^2\big[1 - (4T_b f)^2\big]^2}$$

6.4 Synchronization

- Digital communication system require multiple levels of synchronization

- For a coherent system we need:

  1. Carrier synchronization
  2. Bit/symbol synchronization
  3. Word synchronization

- When we consider spread spectrum later, a spreading code synchronization layer is added

6.4.1 Carrier Synchronization

- The need for a carrier reference can be avoided in most schemes by using either a noncoherent receiver design or in the case of PSK, differential encoding at the transmitter followed by differential decoding at the receiver

- The disadvantage is some loss of power efficiency, i.e., a higher E_b/N_0 is needed to overcome the losses associated with the simplified receiver implementation

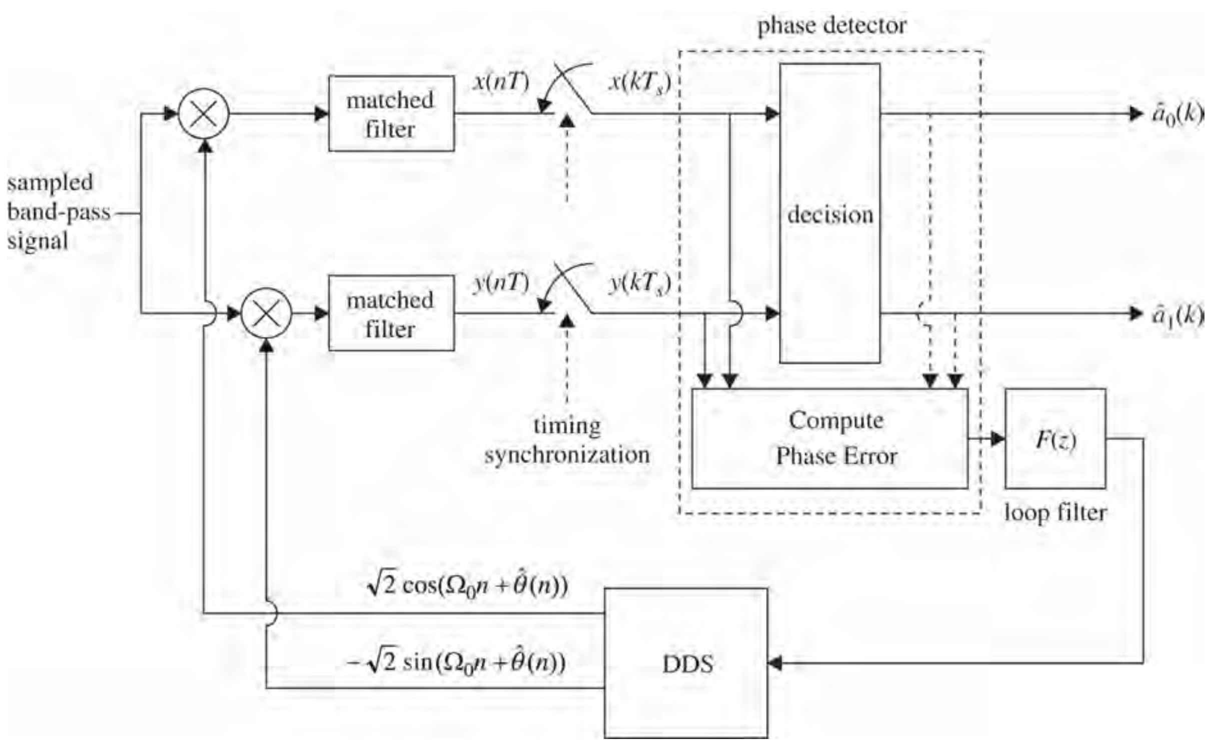
- In most modulation schemes there is no pure carrier present in the transmitted spectrum

- It is possible to add a *pilot carrier* (recall the BPSK modulation index m), but this robs power from the signal and again degrades the E_b/N_0 performance

- When a coherent reference is needed some form of phase tracking subsystem must be implemented

- The Rice text dedicates Chapter 7 to this topic

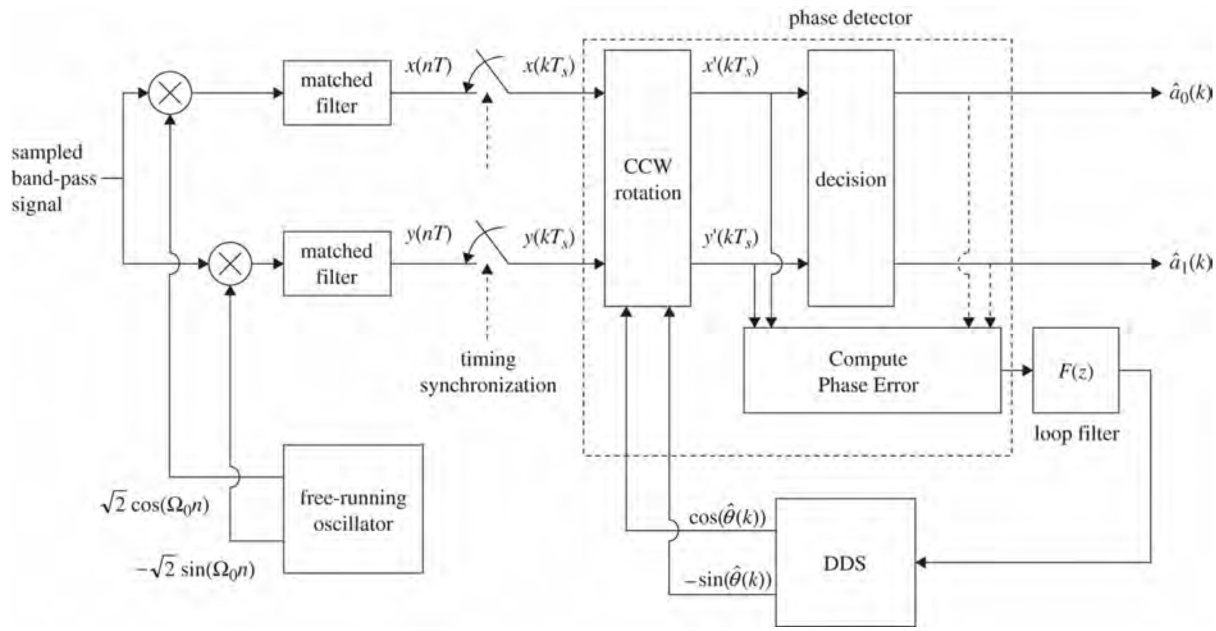
- A high level view of what needs to be accomplished for the case of MQAM is shown below [Rice]:



Complex baseband carrier phase tracking for MQAM

- With this scheme note that the carrier phase is properly rotated into position before the matched filter

- As an alternative, it can be done after the matched filter [Rice]

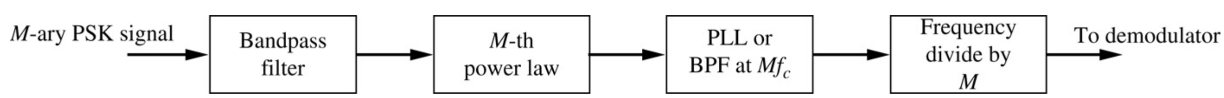


Post matched filter phase rotation

- A key function in any scheme is how to derive phase error information from the data bearing signal

- For PSK one such approach involves a nonlinearity which strips the data modulation from the carrier

- A high level view of this scheme is:



M-power law system for carrier phase synchronization

- A phase-locked loop (PLL) can track the phase of the regenerated carrier, or the phase error generated by other means, such as alluded to in the Rice block diagrams

- To show that it works consider the MPSK signal $s_i(t)$ raised to the Mth power

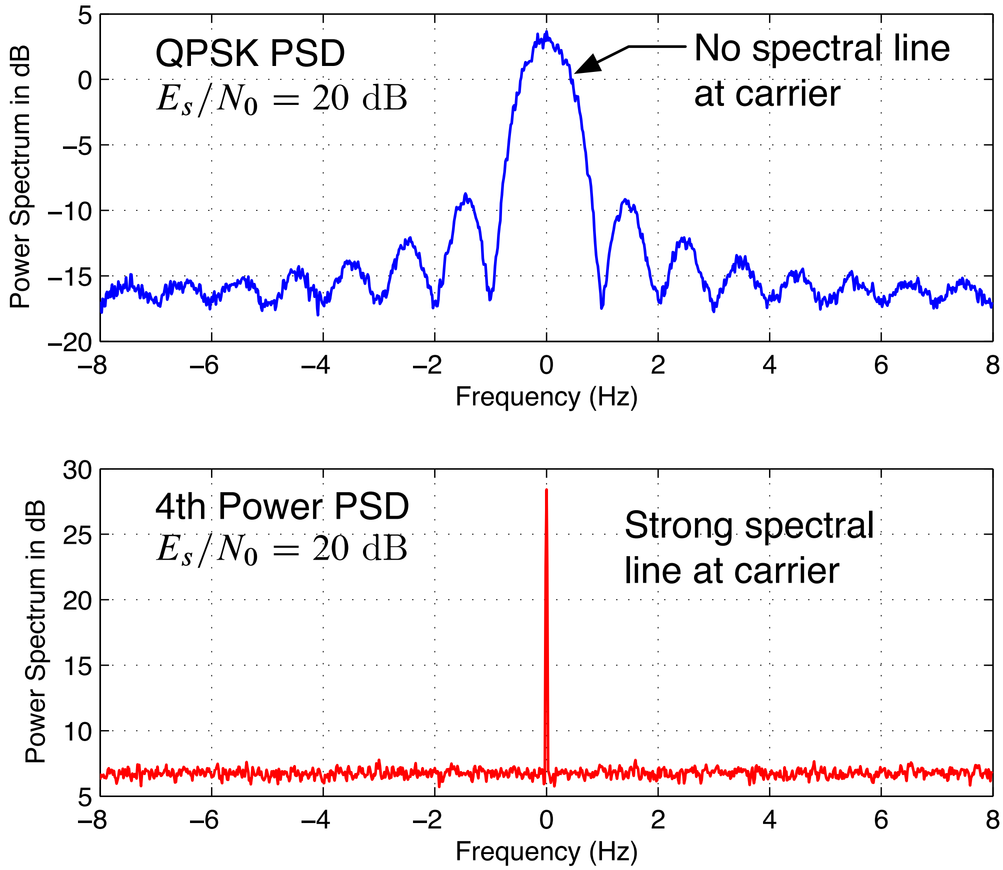
$$
\begin{aligned}
y(t) = \left[s_i(t)\right]^M &= \left[\sqrt{\frac{2E_s}{T_s}}\cos\left(\omega_c t + \frac{2\pi(i-1)}{M}\right) + \theta\right]^M \\
&= \cdots \quad \text{see Z\&T} \\
&= \left(\frac{A}{2}\right)\left\{2\cos(M\omega_c t + M\theta) + \cdots\right\}
\end{aligned}
$$

will contain an unmodulated sinusoidal term of the form $\cos(M\omega_c t)$

Example 6.8: **QPSK**

- Generate a QPSK spectrum and observe the spectrum before and after a 4th-power nonlinearity
- To operating E_s/N_0 was set to 20 dB to give some ideal of the noise floor surrounding the regenerated carrier spectral line
- A portion of this noise enters the carrier tracking loop and results in phase jitter

```
>> x = psk_tx('qpsk',16,10000);
y = cpx_AWGN(x,20,16); % Set Es/N0 to 20 dB
>> subplot(211)
>> spec_plot(y,2^10,16);
>> subplot(212)
>> spec_plot(y.^4,2^10,16,'r');
```



Carrier generation for QPSK

- With noise present, however, tracking the regenerated carrier will result in a random phase jitter or variance, which will impair the SEP (recall the BPSK model with phase error ϕ)

Tracking error variance for BPSK and QPSK

Type of Modulation	**Tracking Error Variance σ_ϕ^2**
None (PLL)	$N_0 B_L / P_c$
BPSK (squaring of Costas loop)	$L^{-1}(1/z + 0.5/z^2)$
QPSK (quadrupling or data est. loop)	$L^{-1}(1/z + 4.5/z^2 + 6/z^3 + 1.5/z^4)$

- The terms in the above table are defined as:

$$\begin{aligned}
T_s &= \text{symbol duration}\\
B_L &= \text{single-sided noise eq. bandwidth of the PLL}\\
N_0 &= \text{single-sided noise PSD}\\
L &= \text{effective number of symb. used in phase est.}\\
&= 1/(B_L T_s)\\
P_c &= \text{signal power (tracked component only)}\\
E_s &= \text{symbol energy}\\
z &= E_s/N_0
\end{aligned}$$

- The above equations are useful in performing system requirements for a carrier phase tracking system

---

Example 6.9: **BPSK Squaring Loop**

- Consider a BPSK link operating with $R_b = 100$ kbps and $E_b/N_0 = 10$ dB
- A squaring loop or Costas loop is configured to have a PLL noise bandwidth of 100 Hz
- Find the tracking error standard deviation in degrees
- From the table

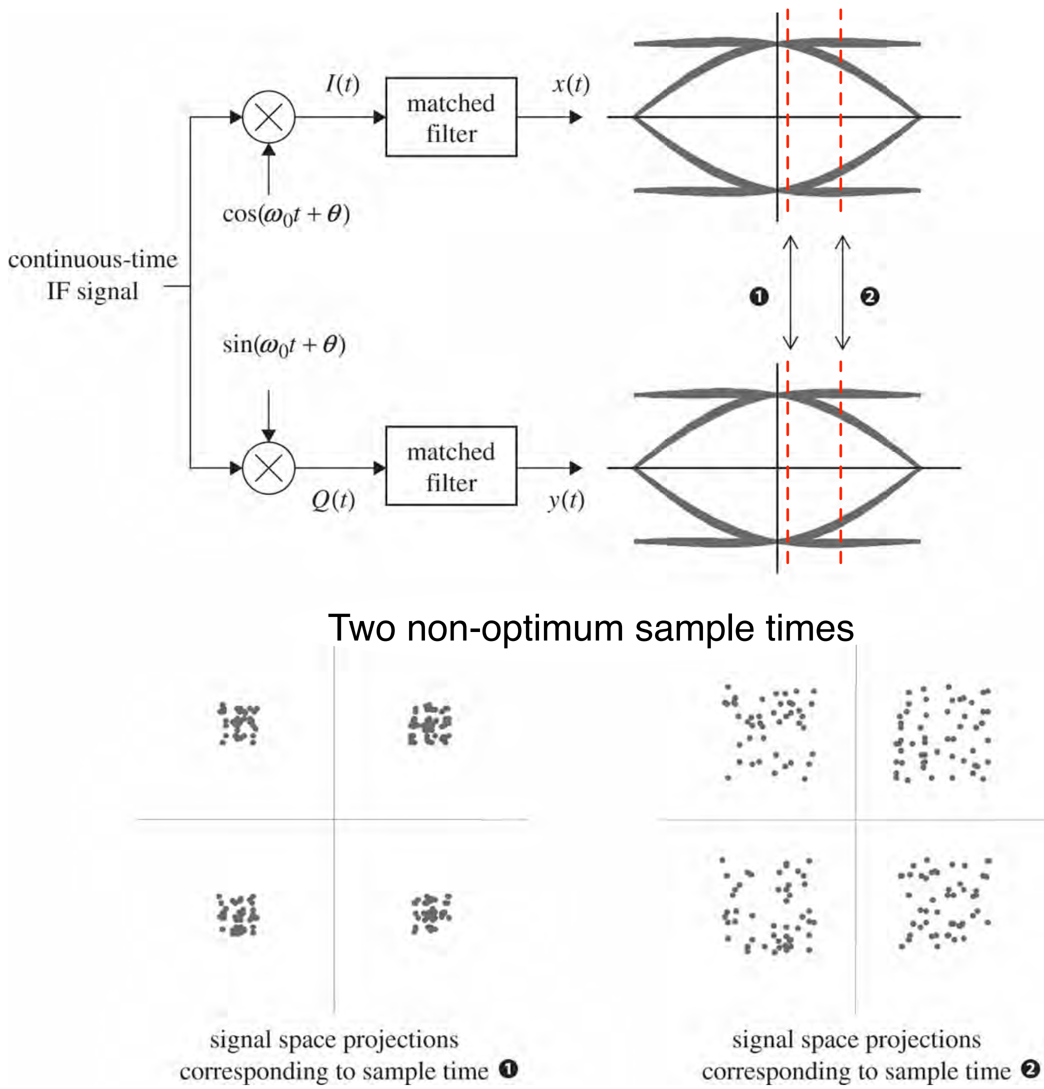
$$\begin{aligned}
\sigma^2_{\phi,\text{Costas}} &= B_L T_b \left(\frac{1}{z} + \frac{1}{2z^2}\right)\\
&= \frac{100}{10^5}\left(\frac{1}{10} + \frac{1}{200}\right) = 1.05 \times 10^{-4}
\end{aligned}$$

so

$$\sigma_{\phi,\text{Costas}} = \sqrt{1.05 \times 10^{-4}} \cdot \frac{180}{\pi} = 0.587^\circ$$

6.4.2 Symbol Synchronization

- The symbol rate clock must be recovered so that the matched filter output can be optimally sampled to drive the decision logic, and produce output symbols (bits)



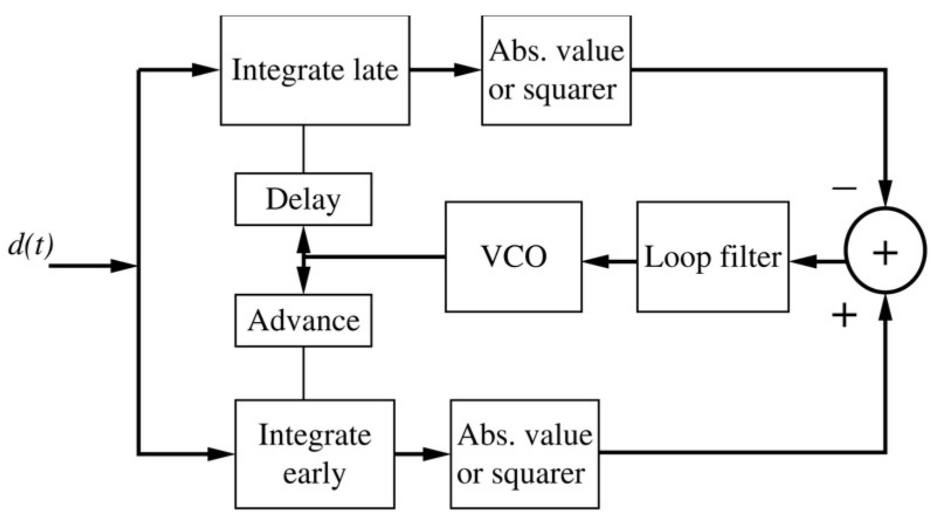
The need for proper symbol timing

- Rice Chapter 8 is dedicated to this topic

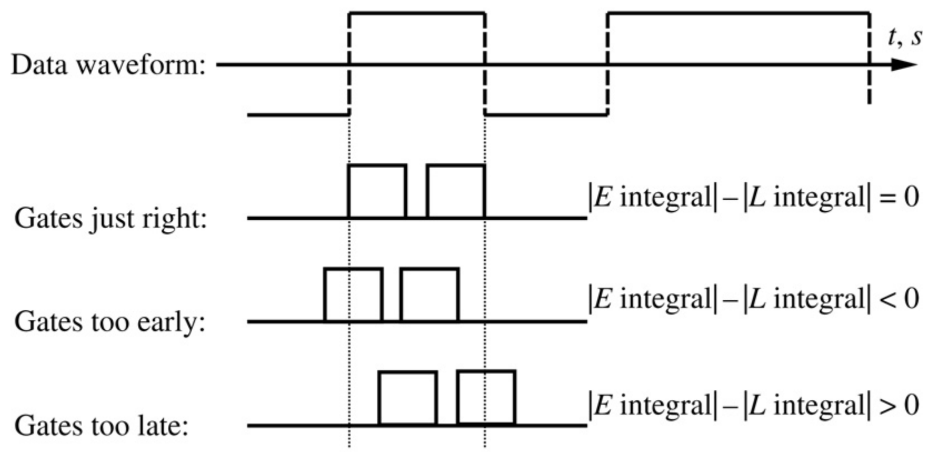
- Three methods to consider are:

  1. Derive the needed clock signal from a primary or secondary standard; both tx and rx may be slaved to a master timing source, e.g., GPS
  2. Use a separate synchronization signal, e.g., pilot clock sent multiplexed on another channel separate from the data, or embedded via line coding
  3. Derive the synchronization from the data bearing waveform itself, i.e., *self-synchronization*

- One traditional synchronizer is the *early-late gate* bit synchronizer



Early-Late Gate Synchronizer

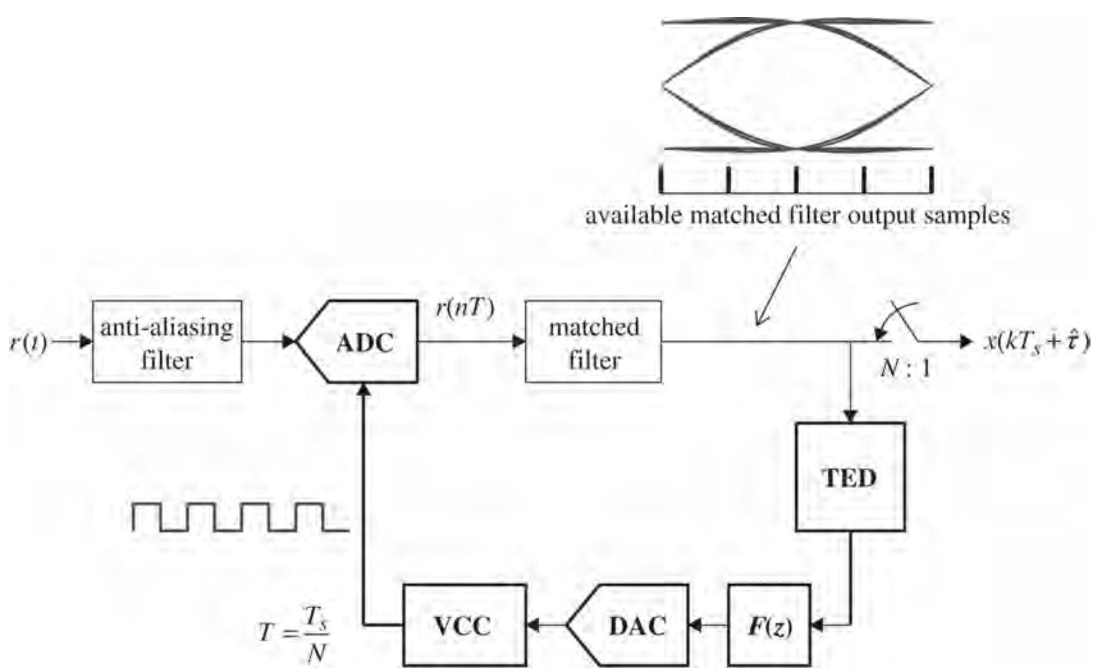


Waveforms

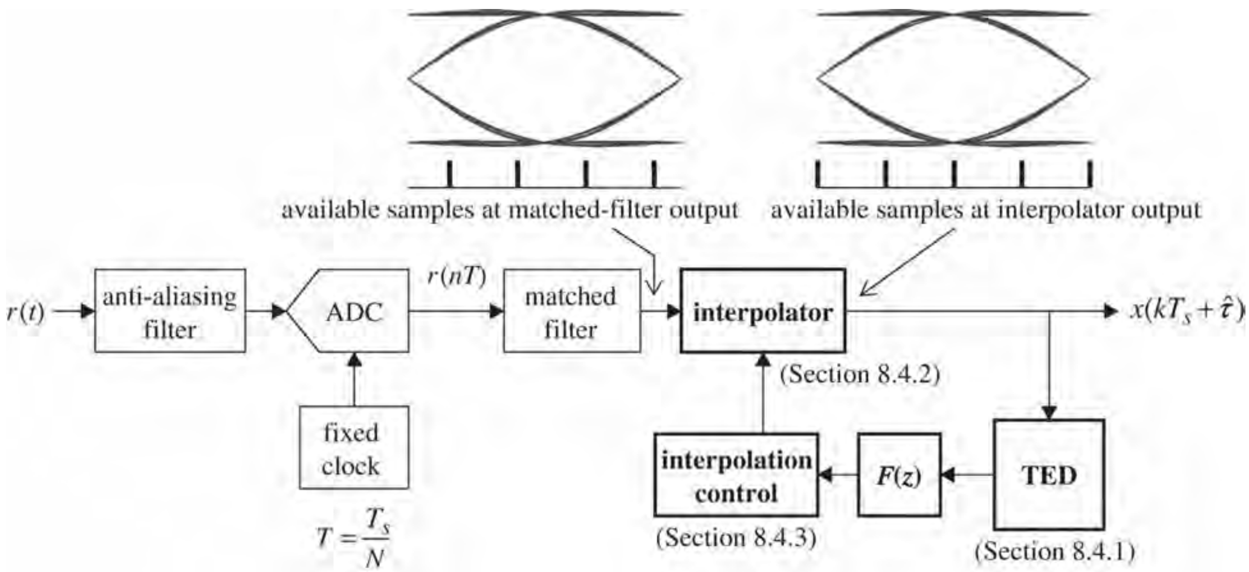
- The timing error variance (jitter) for this type of synchronizer is

$$\sigma^2_{\epsilon,\text{AVG Val}} \simeq \frac{B_L T_B}{8(E_b/N_0)}, \qquad \sigma^2_{\epsilon,\text{SQ-Law}} \simeq \frac{5B_L T_B}{32(E_b/N_0)}$$

- More modern discrete-time approaches are described in Rice

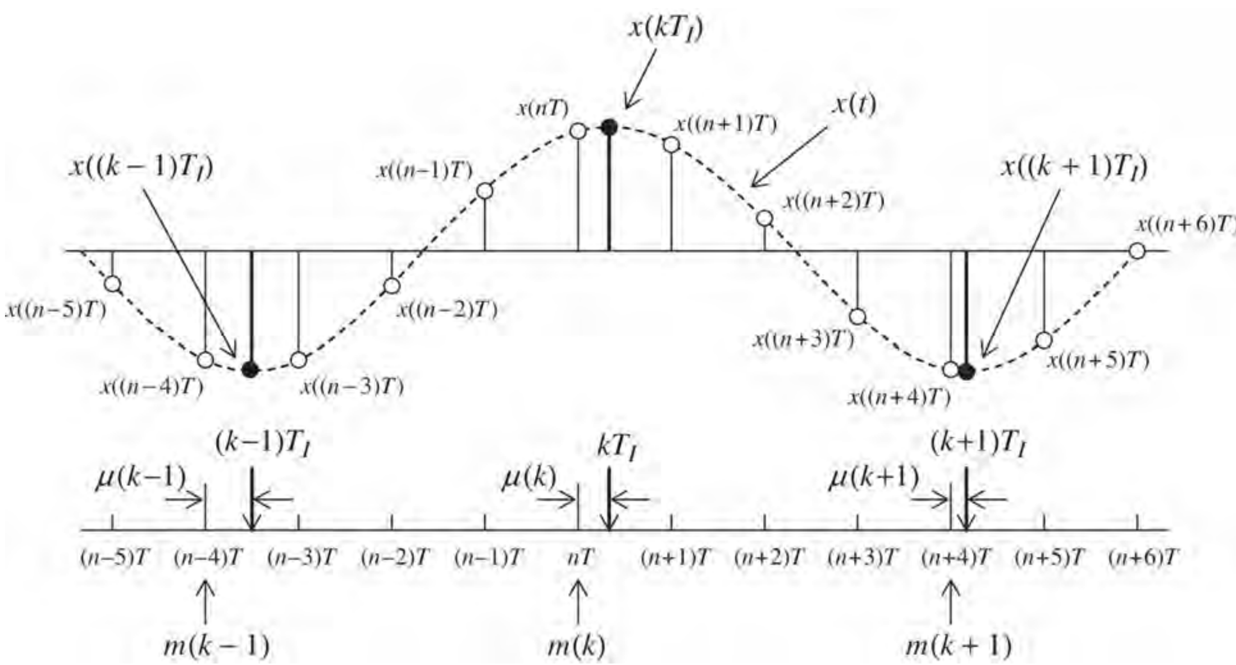


Hybrid continuous/discrete-time signal processing synchronizer

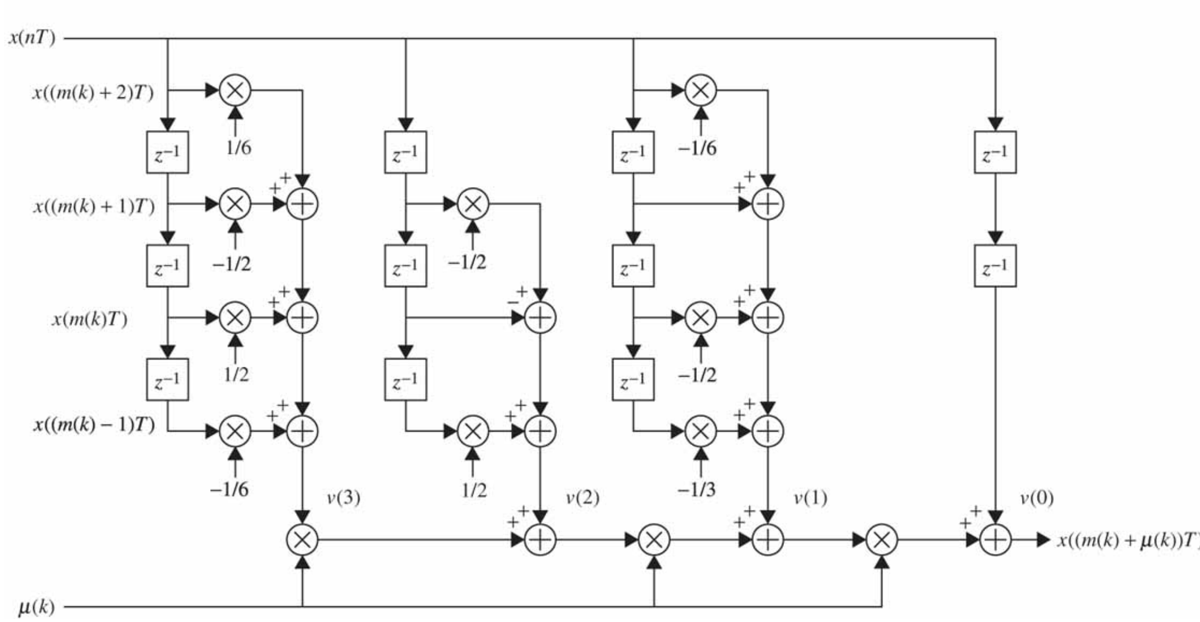


Fixed sampling clock discrete-time synchronizer for PAM signals

- The actions of the *interpolator* are depicted in the waveform below



The actions of the interpolator



Implementation of cubic interpolator using a Farrow structure

6.4.3 Word Synchronization

- Digital data is generally sent in a framing structure
- The options for word synchronization are similar to that of symbol synchronization: (1) primary/secondary standard, (2) a separate synch signal (*marker code*), (3) self-synchronization
- Z&T only discusses (2), stating that the construction of good self-synchronizing codes is not a simple task
- For (2) we have marker codes, which have been designed to have low-magnitude non-zero delay autocorrelation values and and low cross-correlation values with random data

Some good marker codes

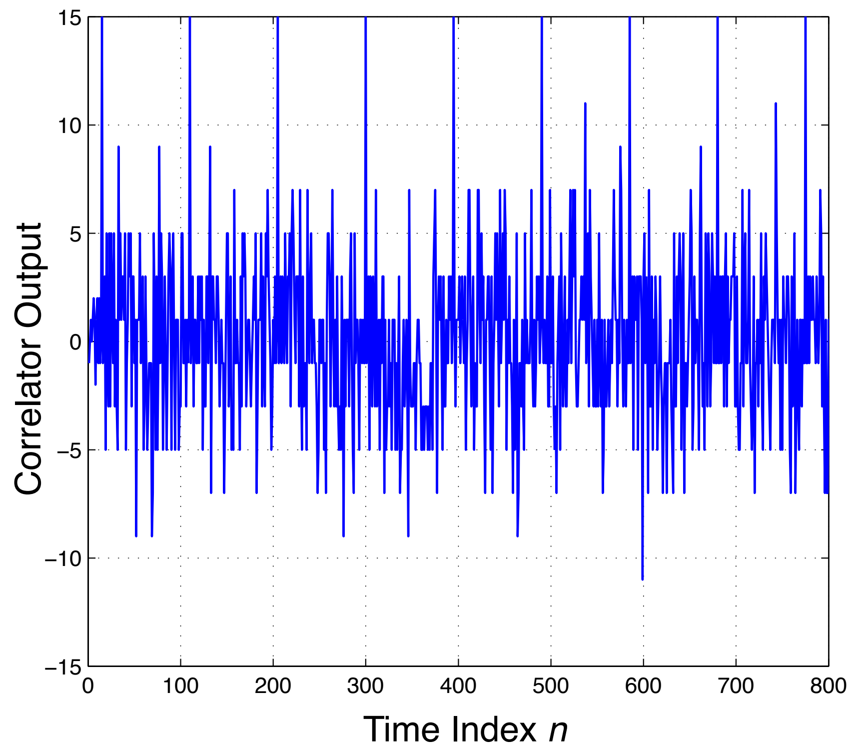
Code	Binary representation	Magnitude: peak correlation*
C7	1 0 1 1 0 0 0	1
C8	1 0 1 1 1 0 0 0	3
C9	1 0 1 1 1 0 0 0 0	2
C10	1 1 0 1 1 1 0 0 0 0	3
C11	1 0 1 1 0 1 1 1 0 0 0	1
C12	1 1 0 1 0 1 1 0 0 0 0 0	2
C13	1 1 1 0 1 0 1 1 0 0 0 0 0	3
C14	1 1 1 0 0 1 0 1 1 0 0 0 0 0	3
C15	1 1 1 1 1 0 0 1 1 0 1 0 1 1 0	3

*Zero-delay correlation = length of code

Example 6.10: Framing Data with Marker Codes

- Two MATLAB functions were written to study the use of the above marker codes as embedded frame synch words
- The function `marker_code` loads the codes given in the above table
- The function `frame_data` inserts N 16-bit words between each copy of an input marker code

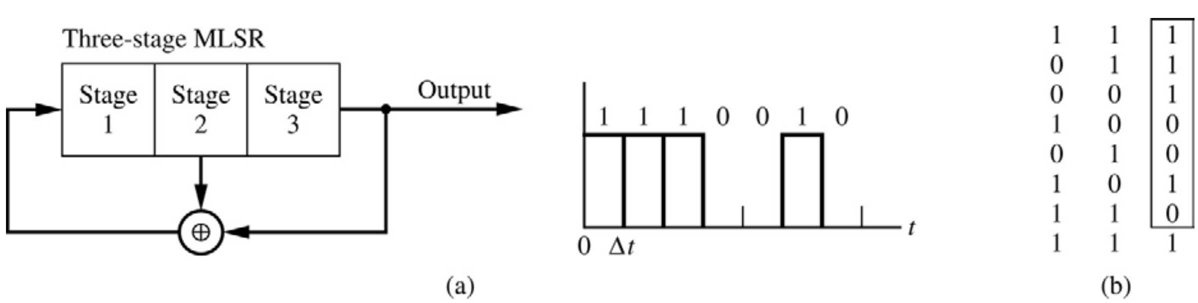
```
>> mc = marker_code('C15');
>> x = randi([0,1],1,1000);
>> y = frame_data(x,5,'C15');
>> z = filter(2*fliplr(mc)-1,1,2*y-1);
>> plot(z)
>> grid
>> n = 0:length(z)-1;
>> axis([0 800 -15 15])
```



Correlator output for $N = 5$ when using the C15 marker code

6.4.4 Pseudo-Noise Sequences

- Pseudo-noise or PN codes are useful in a variety of communications applications
- The sequences are generated using a shift register with an EXOR feedback connection



PN-code generation

- Assuming the 0/1 logic levels in the above sequence generator are transformed to ± 1 waveform levels, the autocorrelation and power spectral density of the code waveform $c(t)$ can be shown to be

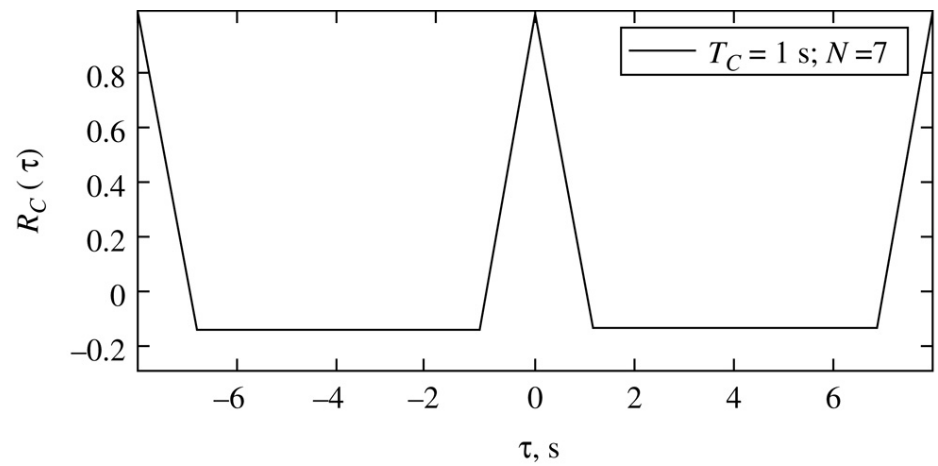
$$R_C(\tau) = \left(1 + \frac{1}{N}\right) \Lambda\left(\frac{\tau}{\Delta t}\right) - \frac{1}{N}, \; |\tau| \leq \frac{N\Delta t}{2}$$

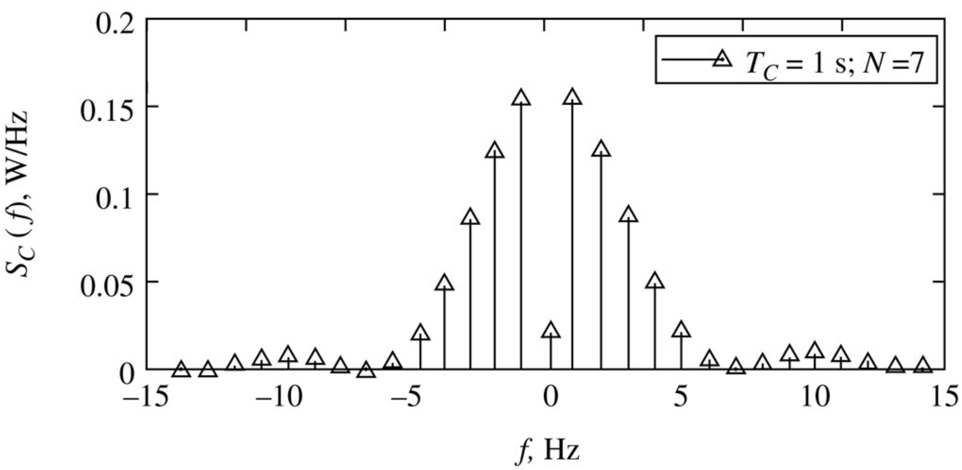
$$S_C(f) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{N+1}{N^2} \text{sinc}^2\left(\frac{n}{N}\right) \delta\left(f - \frac{n}{N\Delta t}\right) + \frac{1}{N^2}\delta(f)$$

- Note: The autocorrelation function is periodic, so the expression above is valid for just the period centered on $\tau = 0$

- Important properties of the PN sequence are:

  - The code period is $2^N - 1$
  - The autocorrelation peak is one with a triangular shape having base width of $2\Delta t$ with minimum (off peak correlation) of $-1/N$
  - The minimum spectral line spacing due to the periodic nature of $c(t)$ is $f_s = 1/(N\Delta t)$
  - Spectral lobes follow a sinc^2 function having zeros or nulls at multiples of $1/\Delta t$, excluding $f = 0$
  - Since the codes have one more 0 (-1) that 1 (+1), the average value of $c(t)$ is $-1/N$, which gives a DC power spike of $1/N^2$ (last term in $S_C(f)$ expression)





Autocorrelation and spectrum properties

- A MATLAB function for generating one period of the sequences

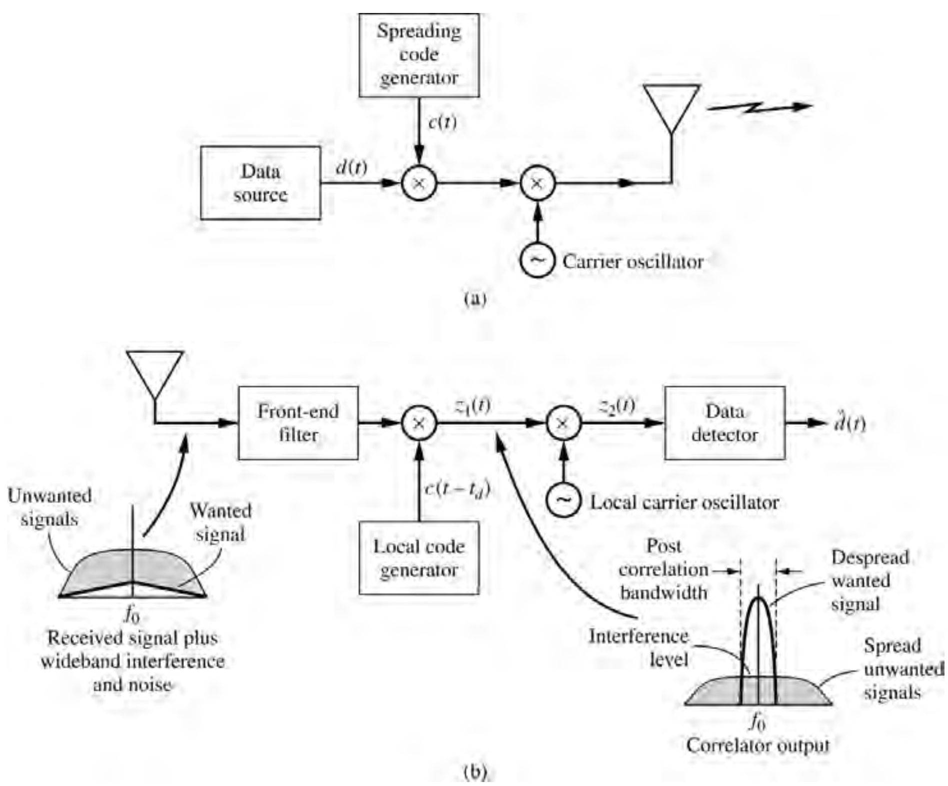
```
function c = m_seq(m)
%function c = m_seq(m)
%
% Generate an m-sequence vector using an all-ones initialization
%
% Mark Wickert, April 2005

sr = ones(1,m);

Q = 2^m - 1;
c = zeros(1,Q);

switch m
    case 2
        taps = [1 1 1];
```

```
    case 3
        taps = [1 0 1 1];
    case 4
        taps = [1 0 0 1 1];
    case 5
        taps = [1 0 0 1 0 1];
    case 6
        taps = [1 0 0 0 0 1 1];
    case 7
        taps = [1 0 0 0 1 0 0 1];
    case 8
        taps = [1 0 0 0 1 1 1 0 1];
    case 9
        taps = [1 0 0 0 0 1 0 0 0 1];
    case 10
        taps = [1 0 0 0 0 0 0 1 0 0 1];
    case 11
        taps = [1 0 0 0 0 0 0 0 0 1 0 1];
    case 12
        taps = [1 0 0 0 0 0 1 0 1 0 0 1 1];
    case 16
        taps = [1 0 1 0 0 0 0 0 0 0 0 1 0 0 0 1 1];
    otherwise
        disp('Invalid length specified')
end

for n=1:Q,
   tap_xor = 0;
   c(n) = sr(m);
   for k=2:m,
       if taps(k) == 1,
           tap_xor = xor(tap_xor,xor(sr(m),sr(m-k+1)));
       end
   end
   sr(2:end) = sr(1:end-1);
   sr(1) = tap_xor;
end
```

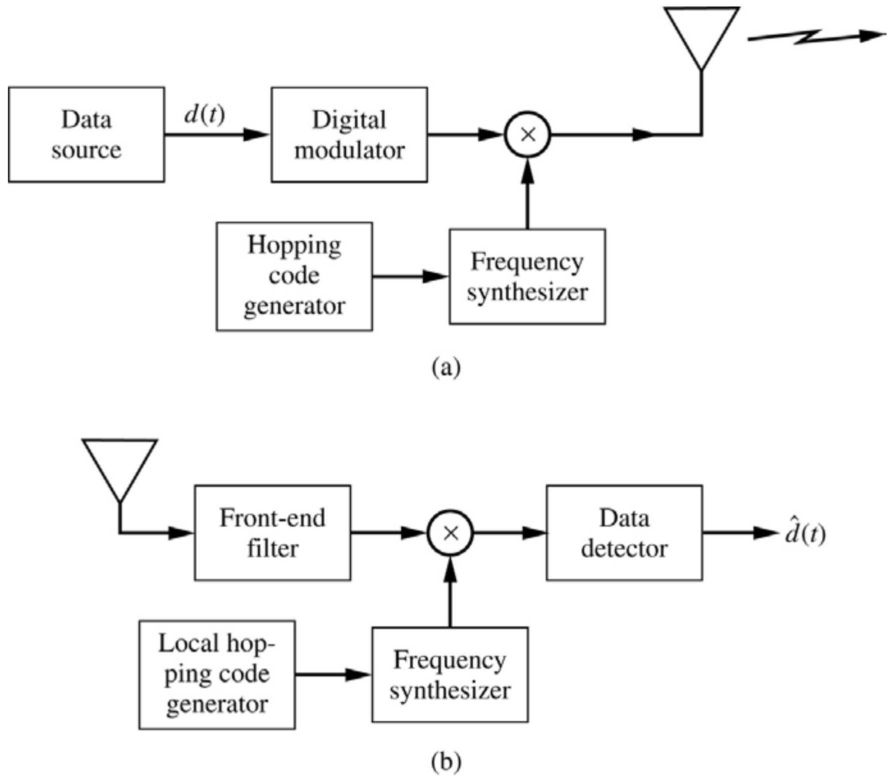
- PN sequences and other families of PN codes are used in ranging systems, such as GPS and can be used a *spreading codes* is spread-spectrum communications (next section)

6.5 Spread-Spectrum Comm Systems

- Spread-spectrum (SS) modulation has the following attributes:
  1. Provide resistance to intentional or unintentional jamming
  2. Provide a means to spread the energy spectral density of the signal out in such a way that it looks like noise, and hence is hard to be intercepted and/or detected by others
  3. Provides resistance to the effects of a multipath channel
  4. Provides a means for more than one user over the same transmission channel (bandwidth)
  5. Provides range measuring capability (GPS, etc.)
- There are two main SS types: *direct sequence* (DS) and *frequency hopping* (FH)



DSSS Transceiver



FHSS transceiver

6.5.1 DSSS

- DSSS generally involves the use of BPSK, QPSK, or MSK
- With BPSK the transmitted signal takes the form

$$x_c(t) = Ad(t)c(t)\cos(\omega_c t + \theta)$$

  where $d(t)$ is the data signal, assumed independent of the spreading code $c(t)$

- The bit rate (denoted chip rate in SS) of $c(t)$ is often an integer multiple of the bit rate of $d(t)$

- Under the independence assumptions, the autocorrelation function of $x_c(t)$ is
$$R_{x_c}(\tau) = \frac{A^2}{2} R_d(\tau) R_c(\tau) \cos(\omega_c \tau)$$
- Assuming rectangular pulse shaping on both sequences
$$R_d(\tau) = \Lambda\left(\frac{\tau}{T_b}\right), \qquad R_c(\tau) = \Lambda\left(\frac{\tau}{T_c}\right)$$
and the corresponding power spectral densities are
$$S_d(f) = T_b \text{sinc}^2(T_b f), \qquad S_c(f) = T_c \text{sinc}^2(T_c f)$$
- The transmitted signal PSD can be obtained via the multiplication theorem for Fourier transforms
$$S_{x_c}(f) = \frac{A^2}{2} S_d(f) * S_c(f) * \mathcal{F}\big[\cos(\omega_c \tau)\big]$$
- Assuming that $R_c = 1/T_c \gg 1/T_b = R_b$, $S_d(f) * S_c(f) \approx S_c(f)$ and we have
$$\begin{aligned} S_{x_c}(f) &= \frac{A^2}{4}\big[S_c(f - f_c) + S_c(f + f_c)\big] \\ &= \frac{A^2 T_c}{4}\left\{\text{sinc}^2\big[T_c(f - f_c)\big] + \text{sinc}^2\big[T_c(f + f_c)\big]\right\} \end{aligned}$$
- The null-to-null bandwidth of a DSSS signal (using rect pulse shapes) is $2/T_c = 2R_c$ centered on the carrier
- In order to recover the data bits at the receiver we need to *despread* the received signal by multiplying by the local spreading code

  - Alignment of the local spreading is a new synchronization layer added with SS

- With noise present as well, the despread signal is

$$z_1(t) = Ad(t)c(t)c(t-\Delta)\cos(\omega_c t + \theta) + n(t)c(t-\Delta)$$

- Under perfect synchronization and downconversion from the carrier using ω_c $\Delta = 0$,

$$z_2(t) = Ad(t) + n'(t)$$

- We can show that $n'(t) = n(t)c(t)\cos(\omega_c t + \theta)$ is a Gaussian process with zero mean and variance at the sampling instant that is $N_0 T_b$

- In the end the BEP for DSSS in AWGN is just

$$P_E = Q\left(\sqrt{\frac{A^2 T_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

6.5.2 DSSS in CW Jamming

- One of the attributes of SS is the ability to mitigate jamming

- A simple form of jamming is single sinusoid (continuous-wave or CW signal) that lies close to the carrier frequency of the desired signal $x_c(t)$

- We model the CW jammer as

$$x_I(t) = A_I \cos\left[(\omega_c + \Delta\omega)t + \phi\right]$$

- At the output of the receiver matched filter (which follows the despreader), we have

$$z_2'(t) = Ad(t) + n'(t) + A_I c(t)\cos(\Delta\omega t + \theta - \phi)$$

- We assume that $\Delta\omega < 2\pi R_c$, that is it lies within the main lobe of the DSSS signal

- The receiver decision statistic now consists of three terms

$$V_0' = \pm AT_b + \underbrace{N_g}_{\text{rv due to noise}} + \underbrace{N_I}_{\text{rv due to jammer}}$$

where

$$N_I = \int_0^{T_b} A_I c(t)\cos(\Delta\omega t + \theta - \phi)\,dt$$

- A Gaussian approximation to N_I is that it is zero mean and has variance

$$\sigma_{N_I}^2 = \frac{T_c T_b A_I^2}{2}$$

- Assuming that N_g and N_I are independent Gaussian rv

$$P_E = Q\left(\sqrt{\frac{A^2 T_b^2}{\sigma_T^2}}\right)$$

where

$$\sigma_T^2 = N_0 T_b + \frac{T_c T_b A_I^2}{2}$$

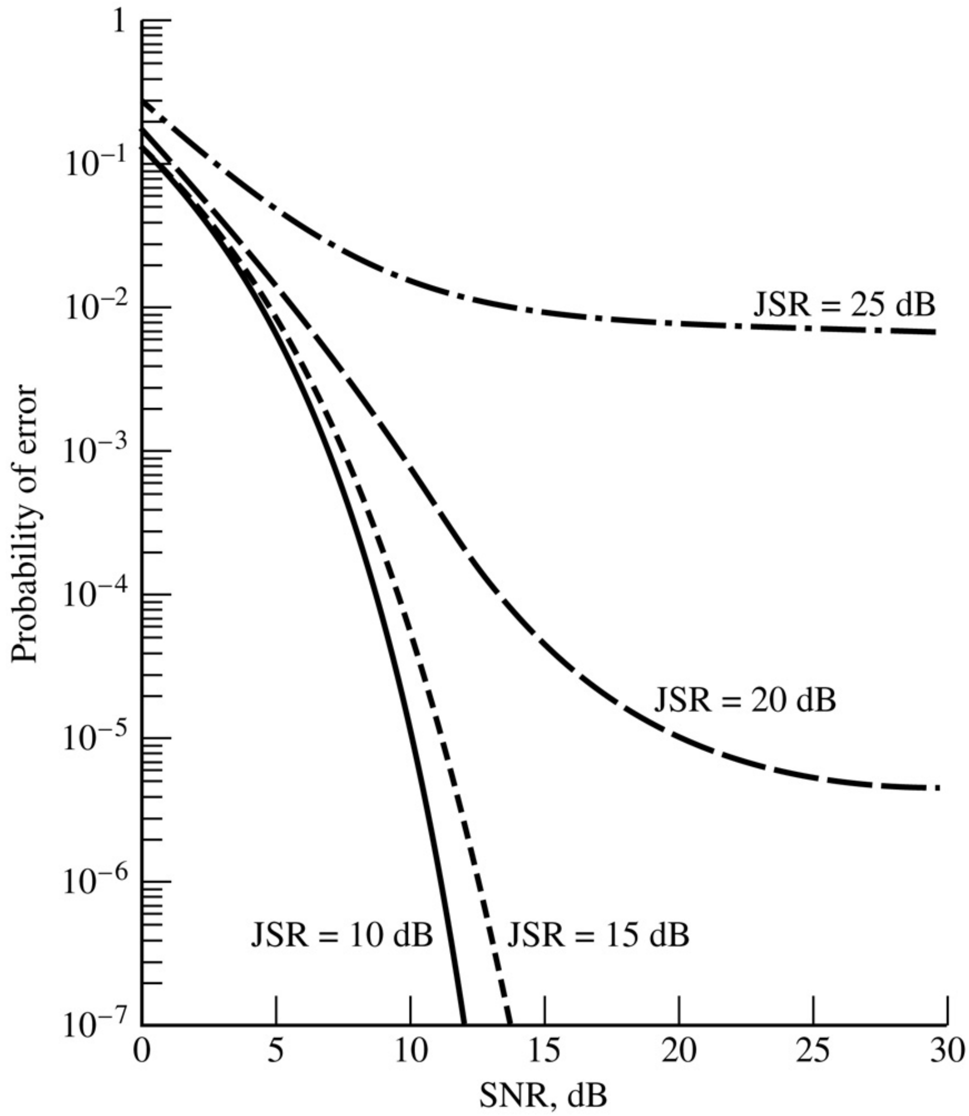
- Manipulation of the Q-function argument yields

$$\frac{A^2T_b^2}{2\sigma_T^2} = \frac{P_s}{P_n + P_I/G_P}$$
$$= \frac{\text{SNR}}{1 + \text{SNR} \cdot \text{JSR}/G_p}$$

  where $P_s = A^2/2$ is the signal power, $P_n = N_0R_b$ is the noise power in a bit rate bandwidth, $P_I = A_I^2/2$ is the jammer power, and $G_p = T_b/T_c = R_c/R_b$ is the DSSS *processing gain* ; in the final line, JSR = P_I/P_s

- When we plot the BEP with JSR as a parameter, we note that an error floor is reached

- A higher processing gain, G_p is needed to overcome the large JSR value



DSSS in CW jamming for $G_p = 30$ dB

6.5.3 Multiple User Environments

- A powerful attribute of DSSS is the ability to overlay multiple signals at the same spectral location, and hence gain frequency reuse
- The scheme is known as *code-division multiple access* (CDMA)

- Each user of a CDMA system is required to have a unique spreading code, approximately orthogonal to the other user codes

- The orthogonality is not perfect and some *multiple access interference* (MAI) exits, thus degrading the BEP

- Very detailed BEP analysis can be preformed, but a simple Gaussian approximation is also possible, within limitations

- Under a Gaussian MAI assumption and perfect power control, that is all signals arriving at the receiver of interest have the same signal strength,

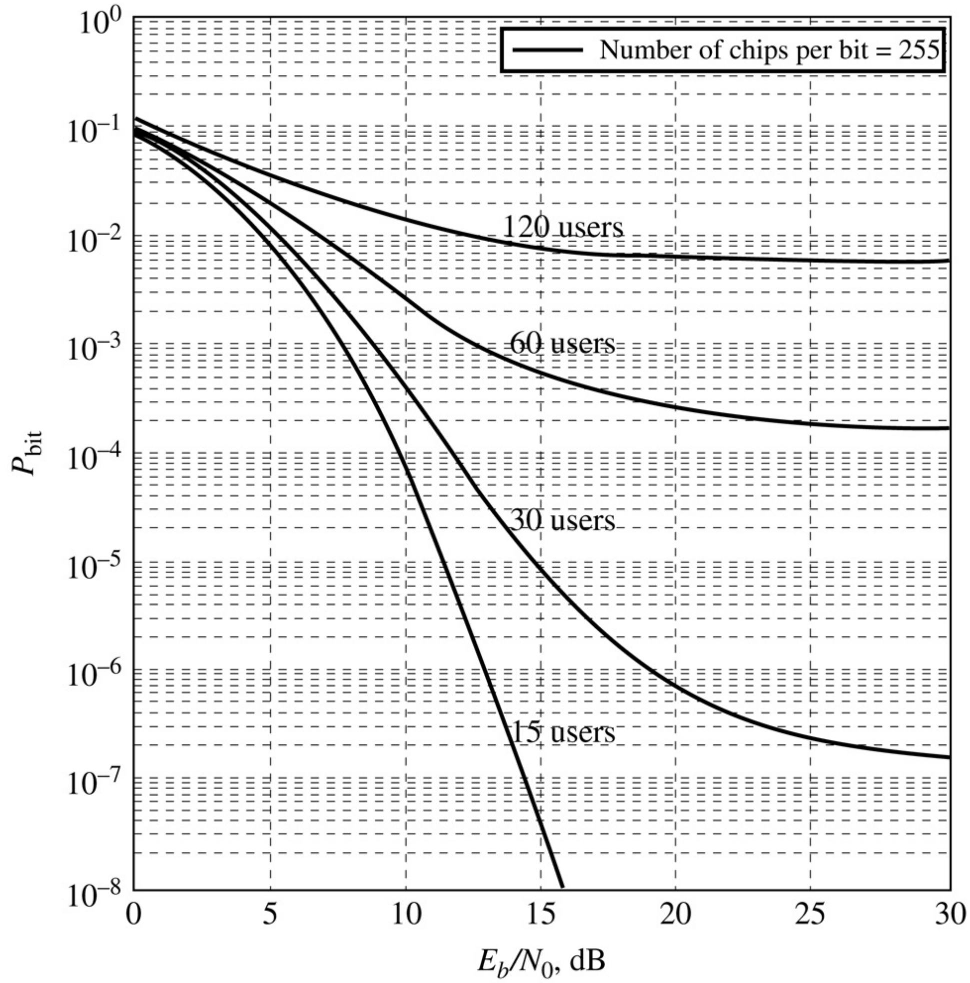
$$P_E = Q\left(\sqrt{\text{SNR}}\right),$$

  where

$$\text{SNR} = \left(\frac{K-1}{3N} + \frac{N_0}{2E_b}\right)^{-1},$$

  in which K is the number of users and N is the number of chips per bit, i.e., the processing gain G_p

- BEP performance is worse when the the arriving powers are not equal, the so-called *near-far* problem means that near (stronger) signals can swamp out far (weak) signals; power control is essential for commercial CDMA systems, e.g., cellular telephony

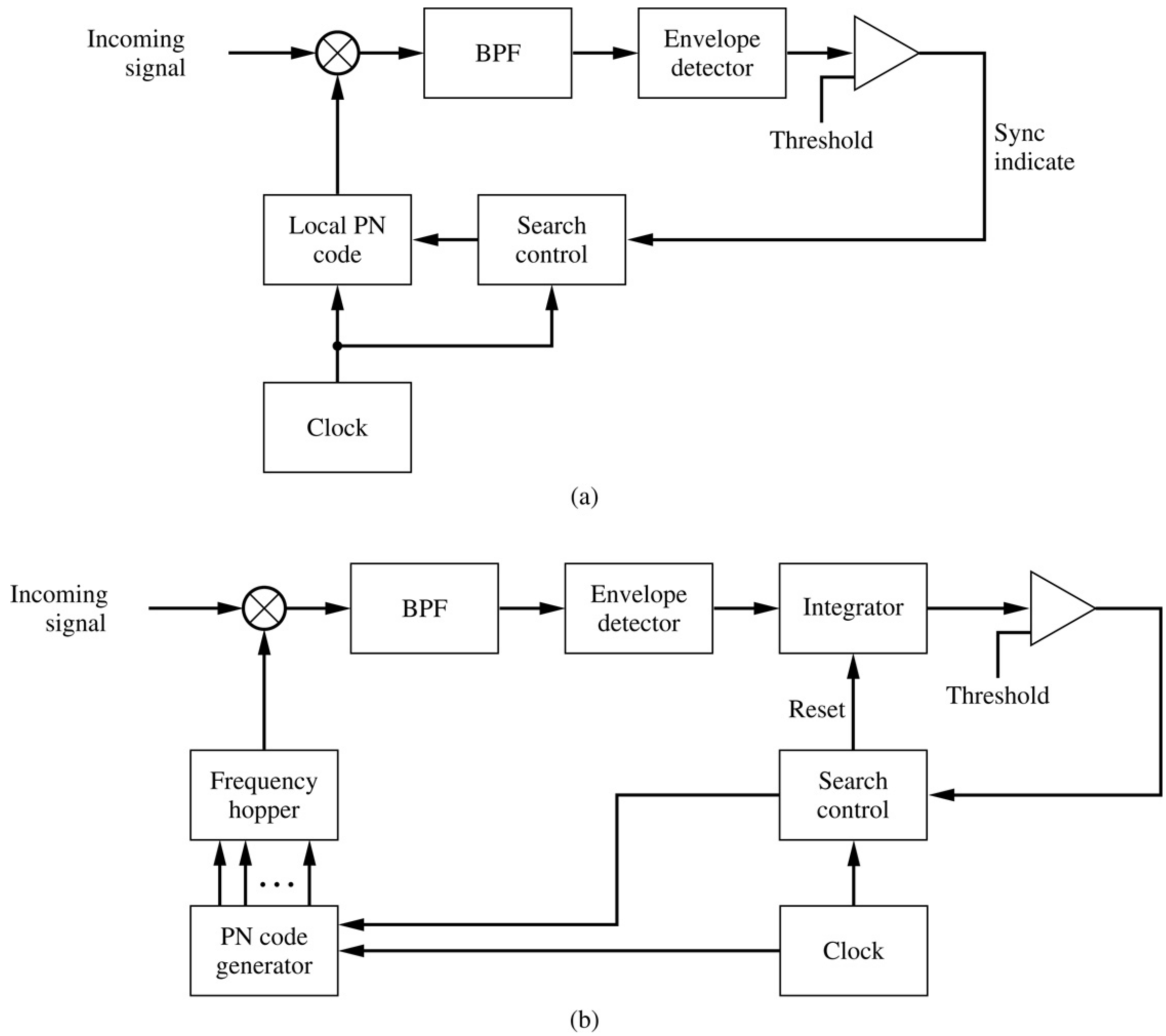


DSSS in MAI

6.5.4 Code Synchronization

- With DSSS and FHSS code synchronization is required
- There are two parts to the code synchronization problem:
  - Code acquisition
  - Code tracking

- For code acquisition serial search is one approach:



Spreading code acquisition and tracking: (a) DSSS, (b) FHSS

- Tracking can begin once $\pm 1/2$ code alignment is achieved
- Serial search commences with the local code being stepped in 1/2 chip increments
  - The code search uncertainty region, C in half-chip increments, may be known a priori
  - Maybe you have to search the entire code period
  - The search pattern can be linear or a zigzag, etc.

- When the acquisition threshold is crossed the codes are approximately aligned and tracking commences

- The mean time to acquisition is given by

$$T_{\mathrm{acq}} = (C-1)T_{\mathrm{da}}\left(\frac{2-P_d}{2P_d}\right) + \frac{T_i}{P_d}$$

where

$$\begin{aligned} C &= \text{code uncertainty region to be searched over} \\ &\quad\ \text{(e.g., in 1/2 chip cells)} \\ P_d &= \text{probability of detection} \\ P_{\mathrm{fa}} &= \text{probability of alarm (false detection)} \\ T_i &= \text{integration time to evaluate each search cell} \\ T_{\mathrm{da}} &= T_i + T_{\mathrm{fa}}P_{\mathrm{fa}} \\ T_{\mathrm{fa}} &= \text{time required to reject an incorrect cell (multiple of } T_i\text{)} \end{aligned}$$

- Note under ideal conditions, $P_d \approx 1$, $P_{\mathrm{fa}} \approx 0$, so the best we can do is

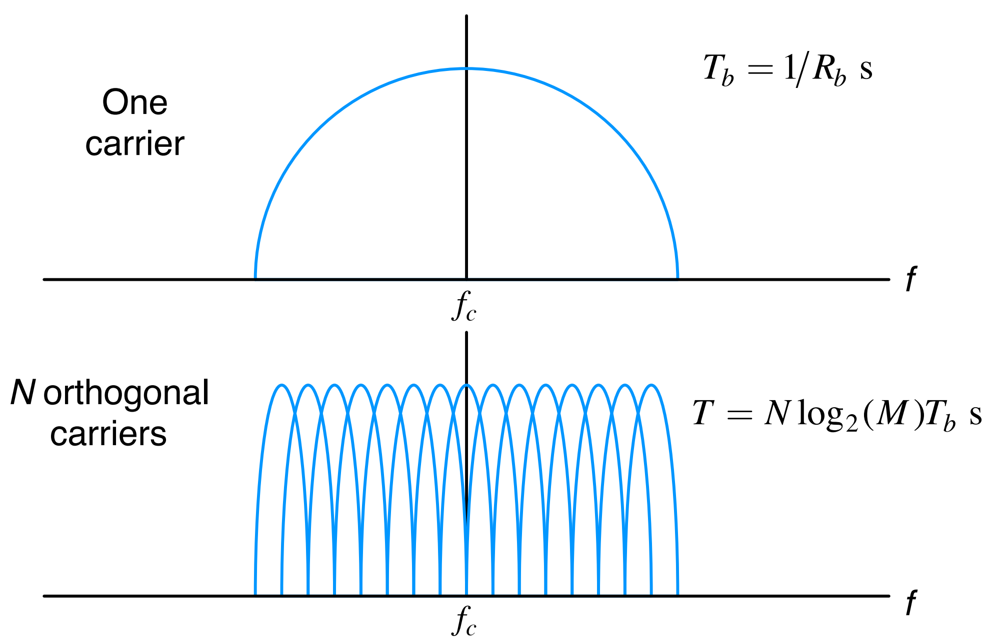
$$T_{\mathrm{acq}} \simeq \frac{C-1}{2}T_i + T_i = \frac{(C+1)T_i}{2} \approx \frac{CT_i}{2}$$

which makes sense; on average we need to search 1/2 the uncertainty interval

- Serial search for FHSS is similar but in this case a frequency pattern is tracked

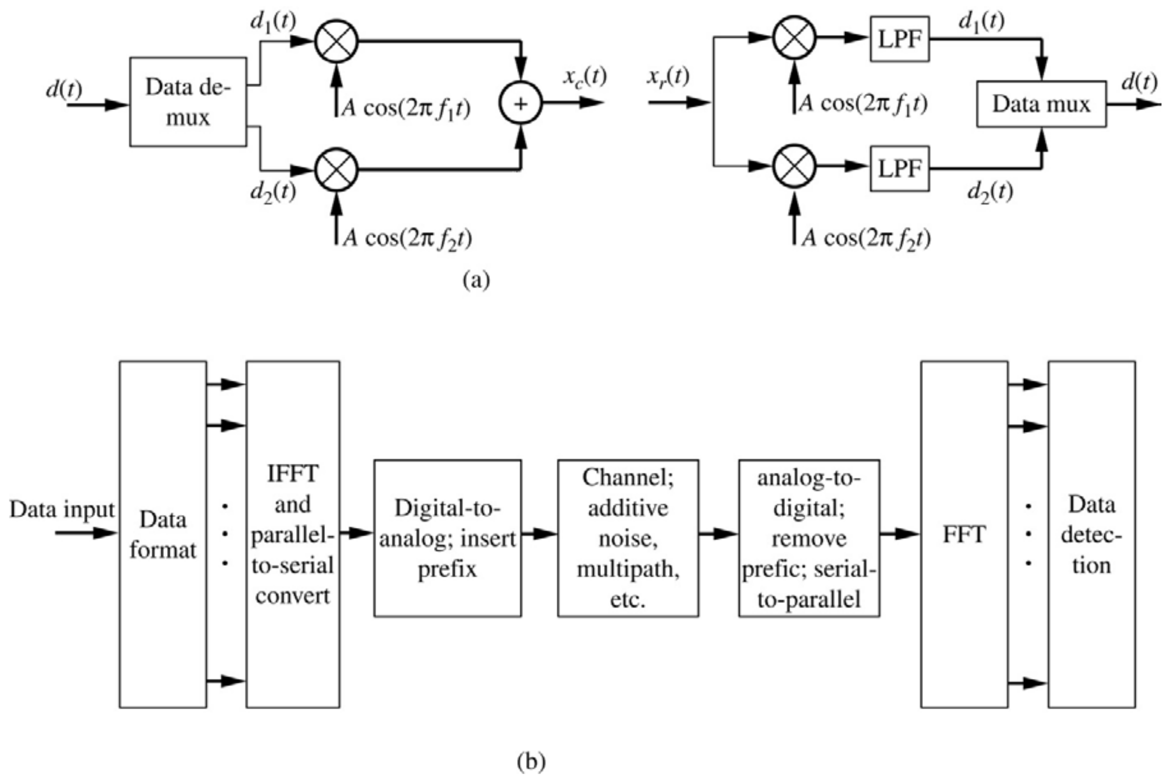
6.6 Multicarrier Modulation and OFDM

- Multicarrier modulation (MCM) and its specialization orthogonal frequency division multiplexing (OFDM), has become quite popular in recent years
  - Digital subscriber lines (DSL)
  - Worldwide Interoperability for Microwave Access (WiMAX), standardized by IEEE 802.16 and WiMAX-Forum
  - Long term evolution (LTE/LTE Advanced) and 4G
- A serial data stream is broken into N subcarrier data streams, each carrying $\log_2(M)$ bits per symbol



Single carrier to N orthogonal carriers

- The implementation can take advantage of IFFT/FFT for transmitter and receiver processing, respectively



MCM and OFDM

- When multi-path which induces ISI is present, the symbol period of each parallel stream can be made much larger than the input period T_b

$$T = \big(N \log_2(M)\big) T_b$$

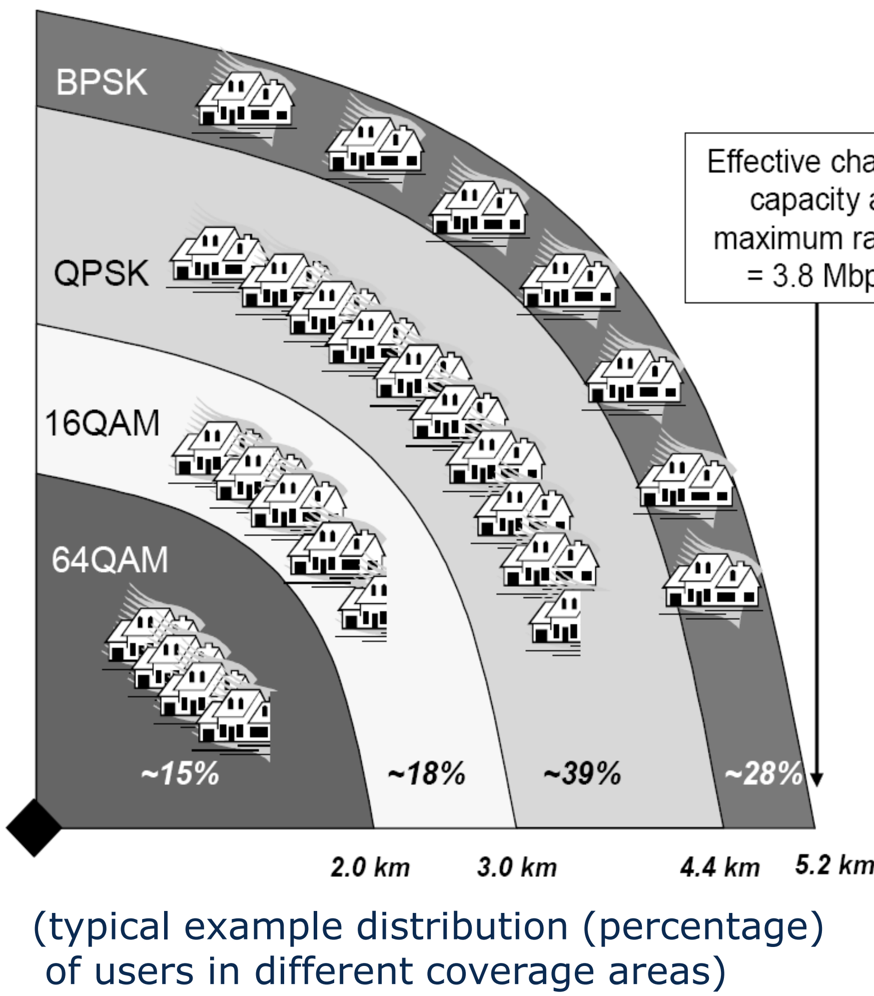
- A *guard time* can be inserted between OFDM symbols to eliminate ISI altogether, almost

- A blank interval will introduce inter-carrier interference (ICI); subcarrier crosstalk

- The solution is to cyclically extend (*cyclic prefix*) the symbol into the guard time so there is an integer number of cycles in the FFT interval

- Subcarrier power levels can be individually adjusted depending upon channel characteristics

- The M-level can be varied depending upon the link SNR (say near or far from the base station)

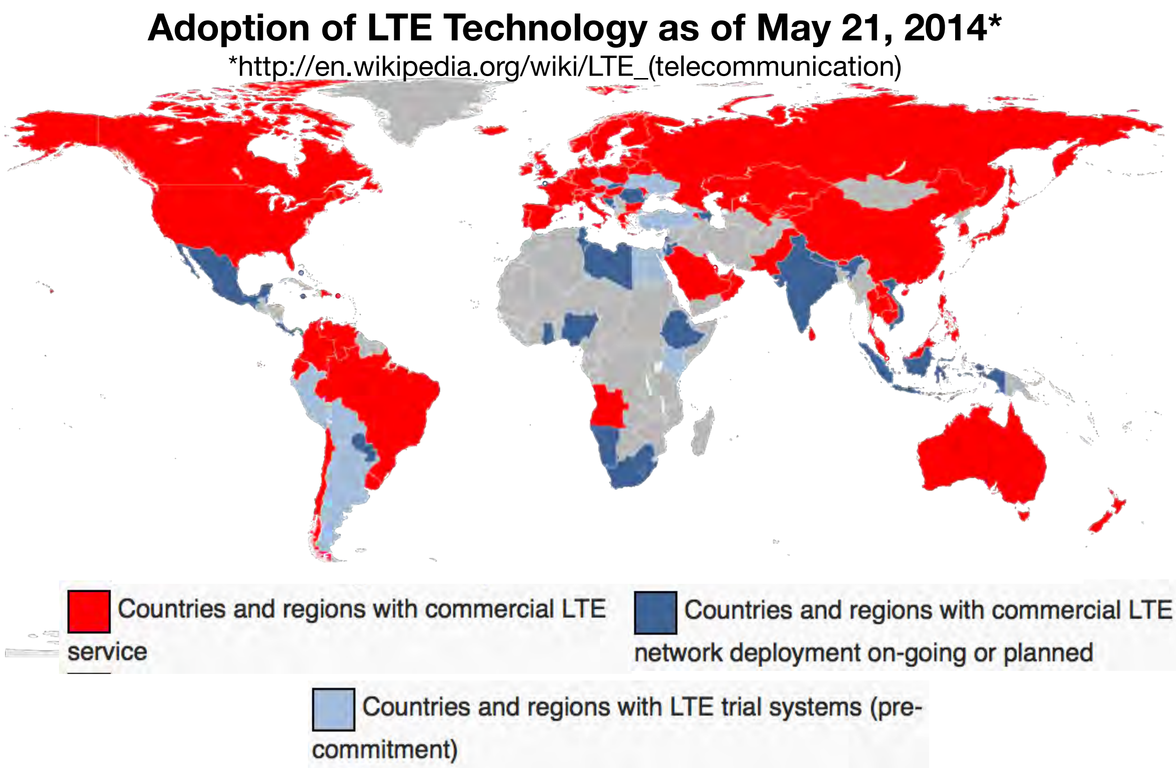


Example of how WiMAX uses M as a function of radial distance from the base station

6.6.1 LTE and OFDM

- Beyond WiMax the 4G standard [2][3]

- LTE with its underlying OFDM modulation is becoming the world standard for cellular communications



Year 2014 adoption of LTE world-wide

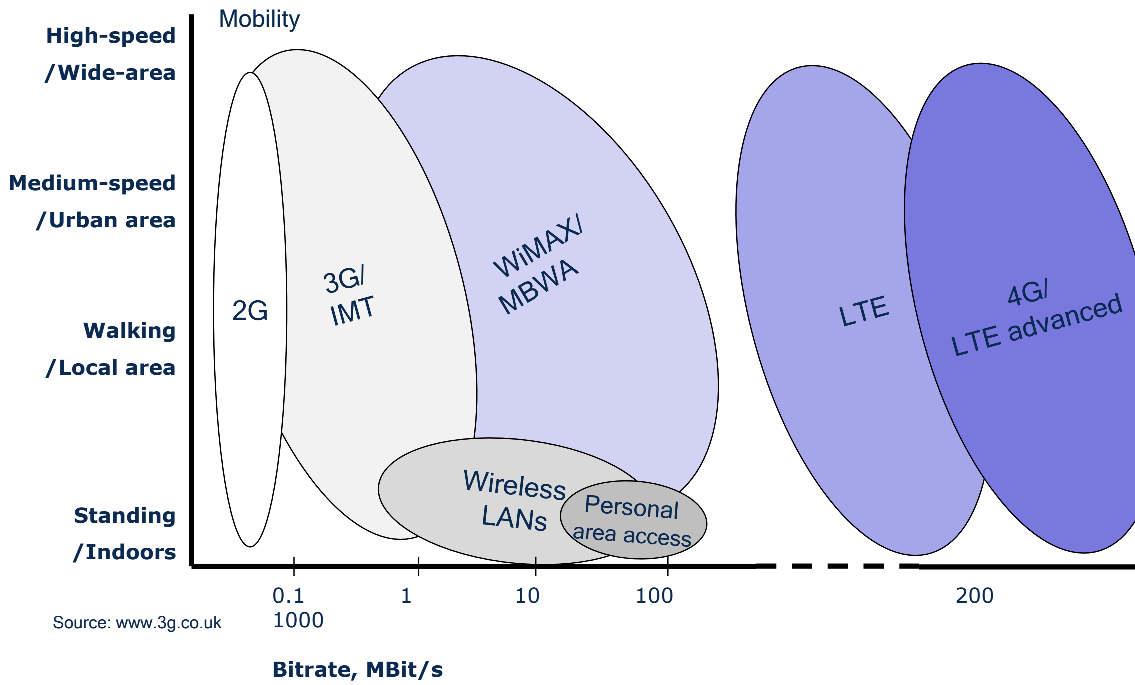
- Data services is the driving factor in all new wireless services

- Wireless voice is of less importance in our lives, so it seems

[2]Sassan Ahmadi, *LTE-Advanced: A Practical Systems Approach to Understanding 3GPP LTE Releases 10 and 11 Radio Access Technologies*, Academic Press, November 14, 2013.

[3]Stefania Sesia (Author), Issam Toufik, *LTE - The UMTS Long Term Evolution: From Theory to Practice*, Wiley, August 29, 2011.

- So how does LTE and LTE Advanced fit into the wireless scene?

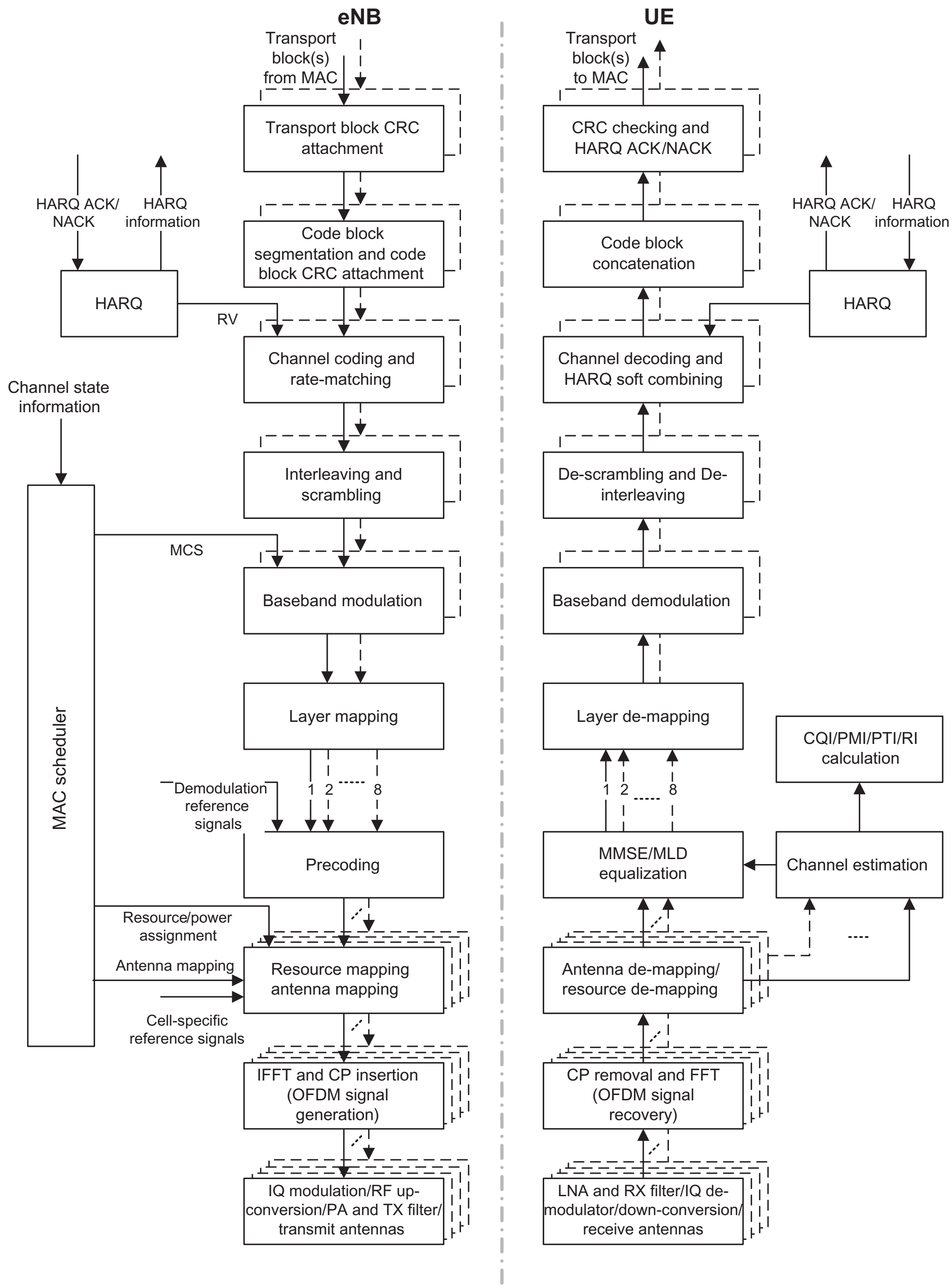


Wireless data compared

- LTE uses pure OFDM for the link from base station to mobile (downlink/forward link) and Single Carrier-Frequency Division Multiple Access (SC-FDMA) for the uplink/reverse link

- The SC-FDMA scheme is used on the uplink for power amplifier efficiency reasons, ... more later

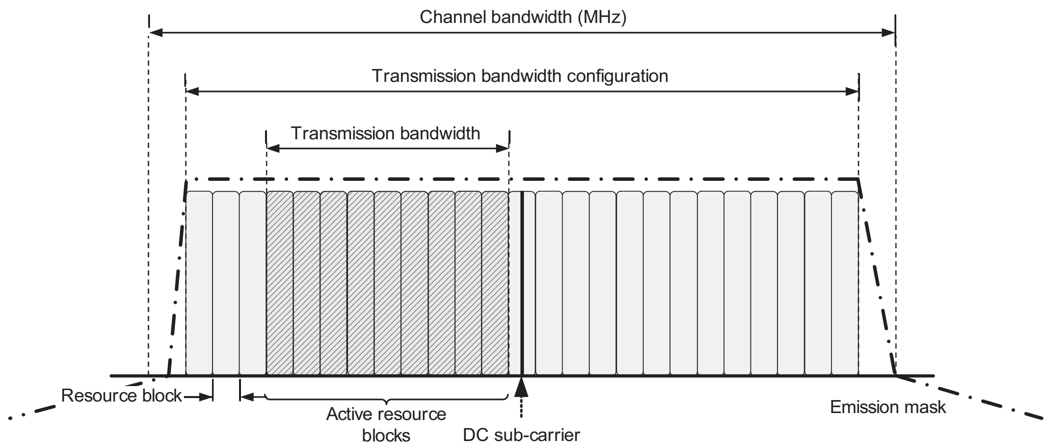
- A big picture view of an LTE transceiver is shown below:



LTE transceiver

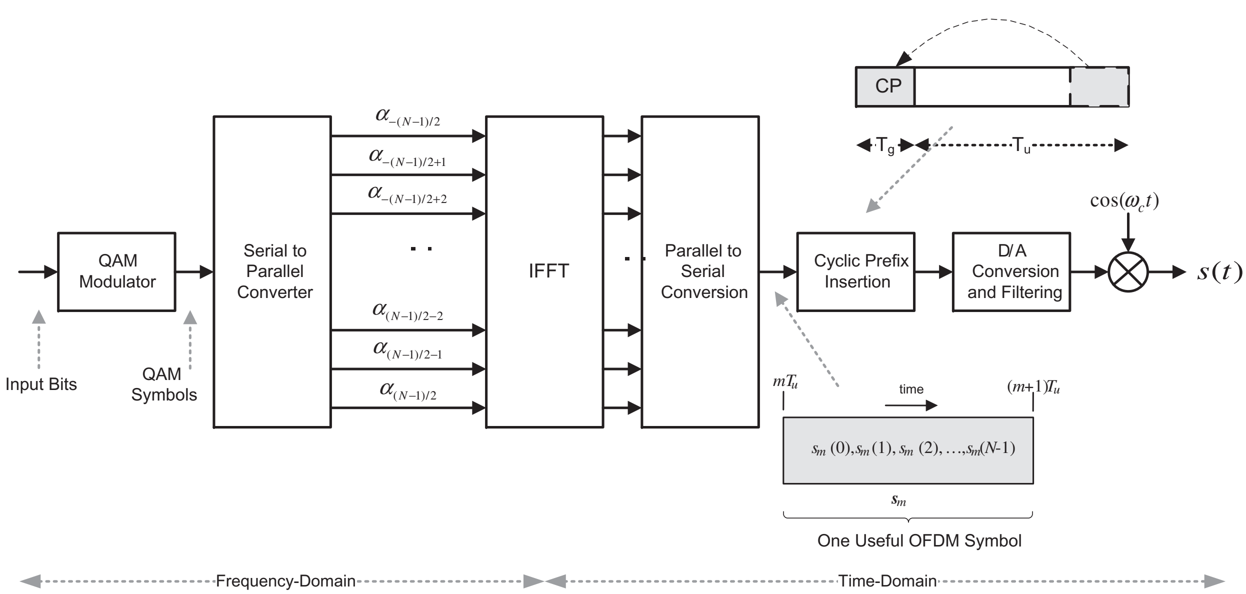
Downlink: OFDM

- The downlink uses OFDM with the frequency layout of the subcarriers as follows



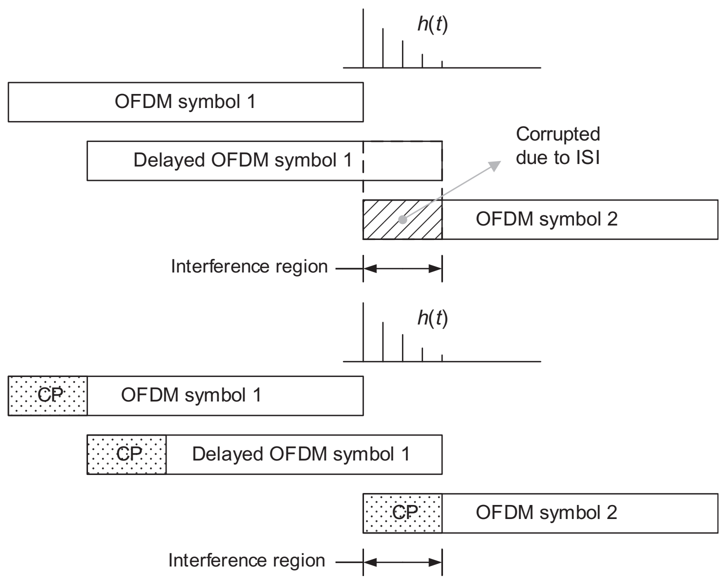
OFDM subcarriers

- The transmitter block diagram



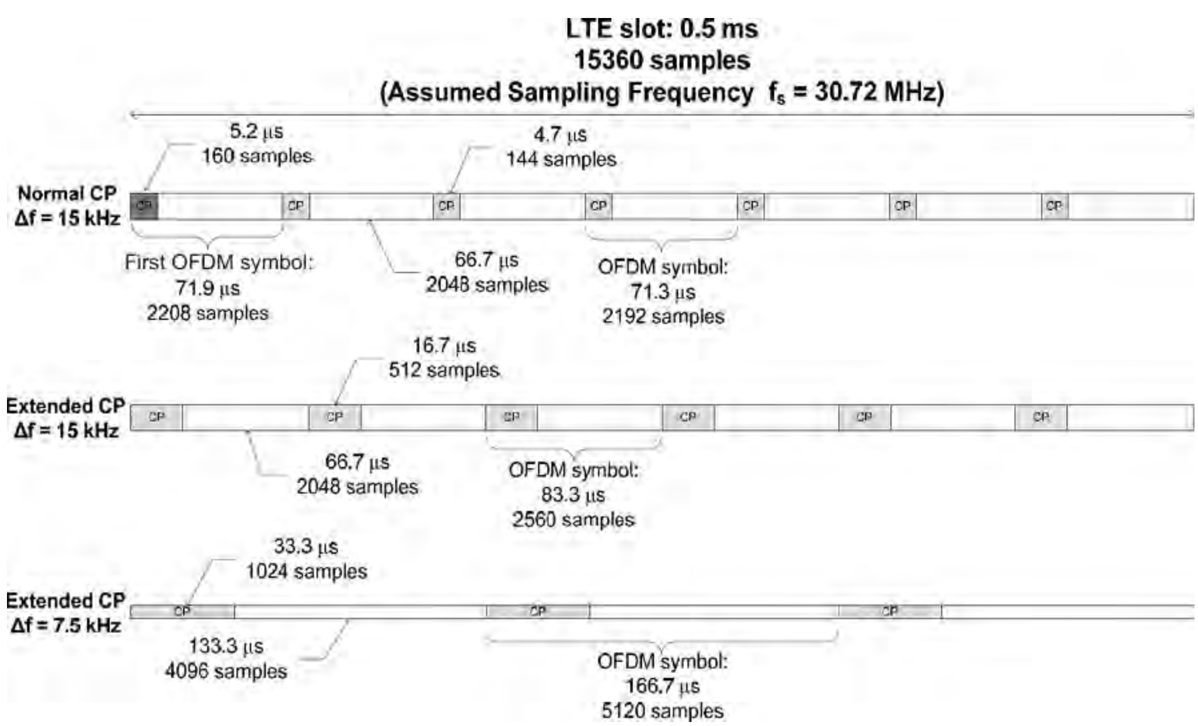
Transmitter block diagram

- The motivation behind the cyclic prefix (CP)



Why the CP works

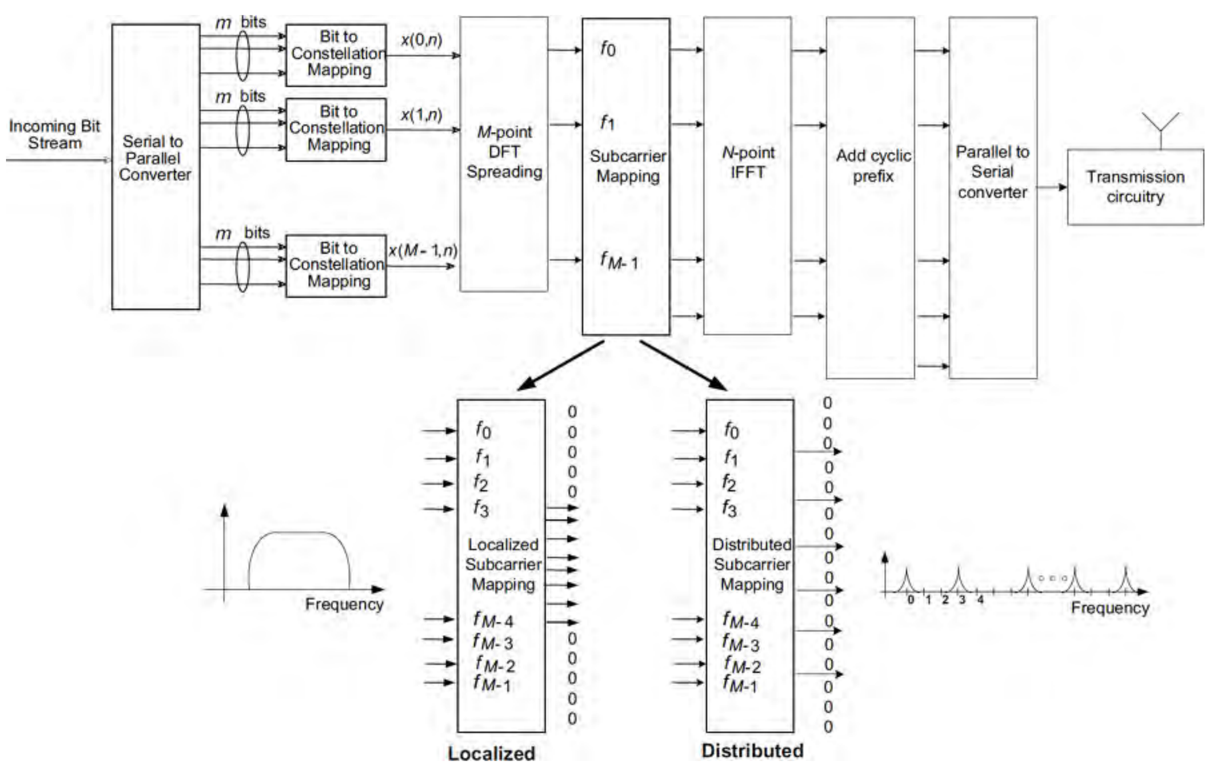
- The LTE slot structure



LTE slot structure

Uplink: SC-FDMA

- OFDM has one downfall; the signal envelope has a rather high peak-to-average ratio (PAR)
- High PAR means that power amplifiers need to be backed off to avoid distortion, but this is inefficient, and shortens battery life on a portable device
- The SC-FDMA scheme used in LTE has a lower PAR and thus is a better choice for the uplink
- From the block diagram below you can see that SC-FDMA looks similar to OFDM



SC-FDMA transmitter signal processing

6.7 Cellular Radio Communications

- Started in the 1970s in both the U.S. and Europe and Japan
- The first commercial system in the US was the *Advanced Mobile Phone System* (AMPS)
  - Used analog FM
  - Channel spacing of 33 kHz
  - At that time data services was not a motivator
- In the 1990s the second-generation (2G) arrived and digital transmission took the place of analog FM
  - In Europe the standard adopted was *Global System for Mobile Communications* (GSM), which used (uses) frequency division multiplexing for multiple access (FDMA)
  - The U.S. and Japan initially adopted time-division multiplexing for multiple access (TDMA)
  - Later the U.S. brought in a second standard (actually an interim standard), IS-95, which uses spread spectrum to achieve code division multiple access (CDMA)
  - In the end GSM became adopted by many countries worldwide; the U.S. maintained multiple standards
- Later in the 1990s work on the third generation (3G) began with the channel allocations in moving from 2G to 3G being simple multiples
- From 2010 onward Fourth-generation (4G) using OFDM matrialized and in 2020 and forward is being replaced by 5G

- As has been the norm since 2G, backward compatibility is being maintained; the handsets have become quite complex multimode radios

- CDMA which became popular in 3G was replaced by OFDM in 4G; even Sprint and Verizon, CDMA champions, moved on to use OFDM in their networks

6.7.1 Cellular Design Foundations

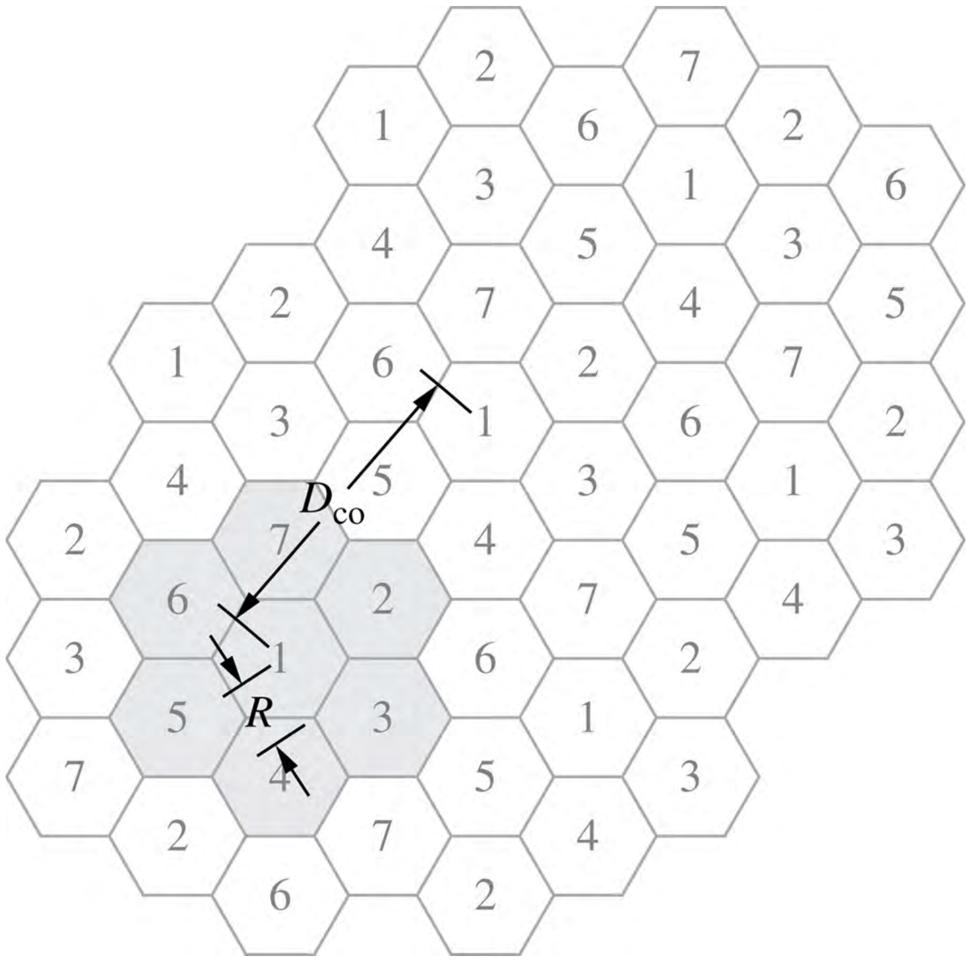
- What defines cellular radio networks is the communications *cell* and the concept of a *reuse pattern* and the *reuse distance*

- The most popular is a seven cell reuse pattern, but IS-95 uses one cell reuse

- The downlink (forward) and uplink (reverse) frequencies assigned for communicating over the reuse pattern are unique, so as to minimize interference

- There are a lot of details to consider when setting up a cellular network

  - At the center of each cell is a cell tower and associated *base station*
  - Collections of base stations connect to the *Mobile Switching Center* (MSC)
  - The MSC ultimately connects to the *Public Switched Telephone Network* (PSTN) to allow all traffic to flow to the global PSTN



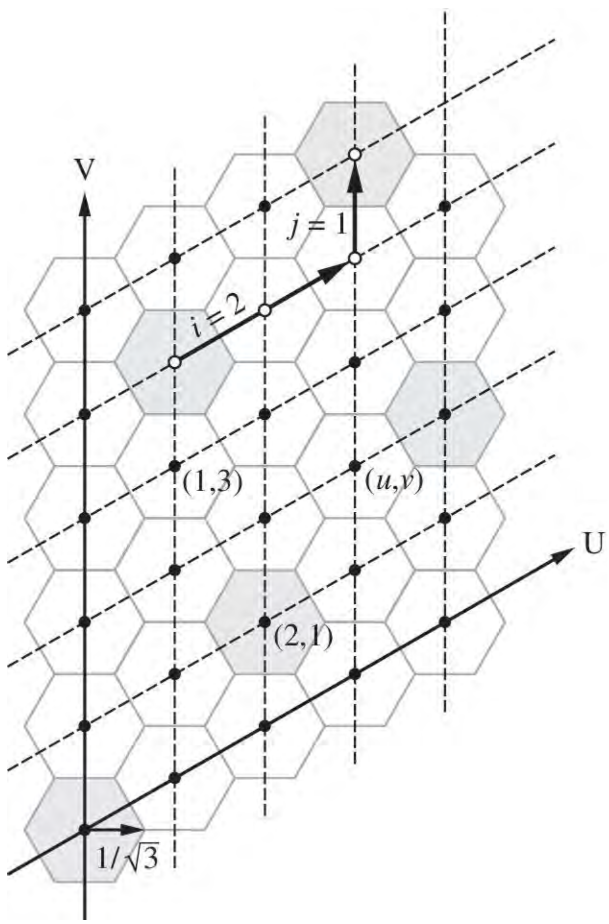
The seven cell reuse pattern with cell radius R and like cell distance D_{co}

- The MSC is responsible for coordinating *handoff* or *handover* as the user moves through the cellular geometry
- Obvious radiowave propagation and transmit signal power levels play a role as well
- The hexagon shape of a cell is used because it is a shape that *tesselates* a plane and approximates a circle; no cells are true hexagons
- In a normalized geometry the cell vertex results in

$$R = 1/\sqrt{3}$$

- If we mark the cell center as having integer coordinates (u, v) we can develop formulas for the number of cells in a reuse patterns and the reuse distance

- It can be shown that the number of cells, N, in a reuse pattern is
$$N = i^2 + ij + j^2$$
where i and j are integers that define the coordinate shift required to arrive at the first reuse cell



Grid geometry and coordinates of the seven cell pattern

- In the figure above we see that moving over by coordinate shift

$i = 2$ and up by coordinate shift $j = 1$ gives $N = 2^2 + 2 + 1 = 7$ Common reuse pattern sizes include:

- IS-95/cdmaOne has $N = 1$
- Original AMPS has $N = 7$
- GSM has $N = 12$

- The distance between like-cell centers, $D_{\mathbf{co}}$, is useful in *cochannel interference analysis*, and can be shown to be

$$D_{\mathbf{co}} = \sqrt{3N}R = \sqrt{N}$$

- **Note:** A maximum of six cochannel users surround the user, and this is just the first *ring*

Cell reuse patterns for various i and j values

Reuse Coordinates i	j	Number of Cells in Reuse Pattern N	Normalized Distance Between Repeat Cells $\sqrt{N}$
1	0	1	1
1	1	3	1.732
1	2	7	2.646
2	2	12	3.464
1	3	13	3.606
2	3	19	4.359
1	4	21	4.583
2	4	28	5.292
1	5	31	5.568

6.7.2 Simple Cochannel Interference Analysis

- In free space the received signal power is inverse proportional to the square (power law is 2) of the distance from the transmitter

- In terrestrial propagation the power law is in the range of 2.5 to 4, that is the received power as distance r can be modeled as

$$P_r(r) = K\left(\frac{r_0}{r}\right)^{\alpha} \textbf{ watts}$$

where r_0 is the reference distance and K is the known power in watts

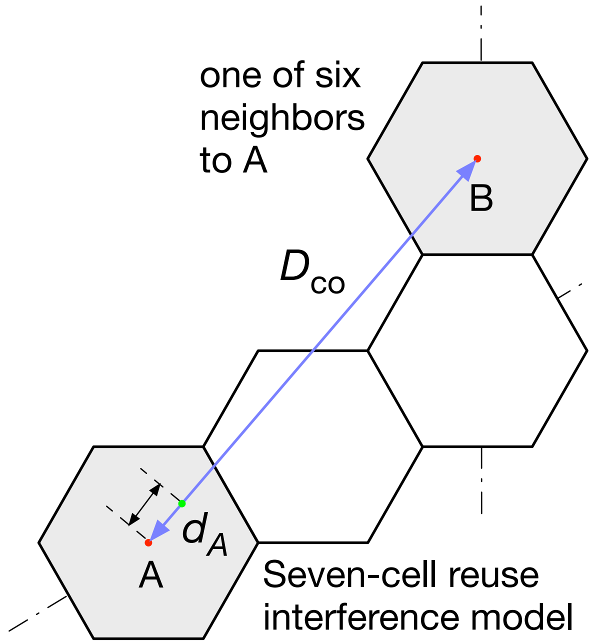
  - **Note:** $\alpha > 2$ is due in part to the earth's surface is acting as a partially conducting reflector; scattering due to large objects (e.g., buildings) also comes into play

- Expressed in dB units we have

$$P_{r,\text{dBW}}(r) = K_{\text{dBW}} + 10\alpha\log_{10} r_0 - 10\alpha\log_{10} r \text{ dBW}$$

where $K_{\text{dBW}} = 10\log_{10} K$

- Using this propagation model we will now consider the mobile reception power at distance d_A from the base station and the interference power from a cochannel base station B at distance $D_{\mathbf{co}}$ from A



Interference analysis cell geometry

- We can now write an expression for the signal-to-interference ratio (SIR) at the mobile station assuming equal power K from each base station:

$$\begin{aligned}\mathrm{SIR_{dB}} &= K_{\mathrm{dBW}} + 10\alpha\log_{10} r_0 - 10\alpha\log_{10} d_A \\ &\quad - \left[K_{\mathrm{dBW}} + 10\alpha\log_{10} r_0 - 10\alpha\log_{10}\left(D_{\mathrm{co}} - d_A\right)\right] \\ &= 10\alpha\log_{10}\left(\frac{D_{\mathrm{co}} - d_A}{d_A}\right) = 10\alpha\log_{10}\left(\frac{D_{\mathrm{co}}}{d_A} - 1\right)\ \mathrm{dB}\end{aligned}$$

- We now have a means of predicting the SIR as the mobile moves from the cell center ($d_A = 0$) to the cell edge ($d_A = R$) along the line connecting A and B

- For six cochannel interferers the lower bound on SIR is

$$\mathrm{SIR_{dB,\,min}} = 10\alpha\log_{10}\left(\frac{D_{\mathrm{co}}}{d_A} - 1\right) - 10\log_{10}(6)\ \mathrm{dB}$$

Note: This worst case because from the above figure we see that only of of the cochannel cells has the mobile as close as $D_{\text{co}} - d_A$ to the interfering cell

- At the cell edge for example

$$\frac{D_{\mathbf{co}}}{d_A} = \frac{\sqrt{3N}\,R}{R} = \sqrt{3N}$$

so the SIR becomes

$$\text{SIR}_{\text{dB, min, edge}} = 10\alpha \log_{10}\left(\sqrt{3N} - 1\right) - 7.7815 \text{ dB}$$

- See text Examples 10.9 and 10.10 for an interesting cell efficiency analysis

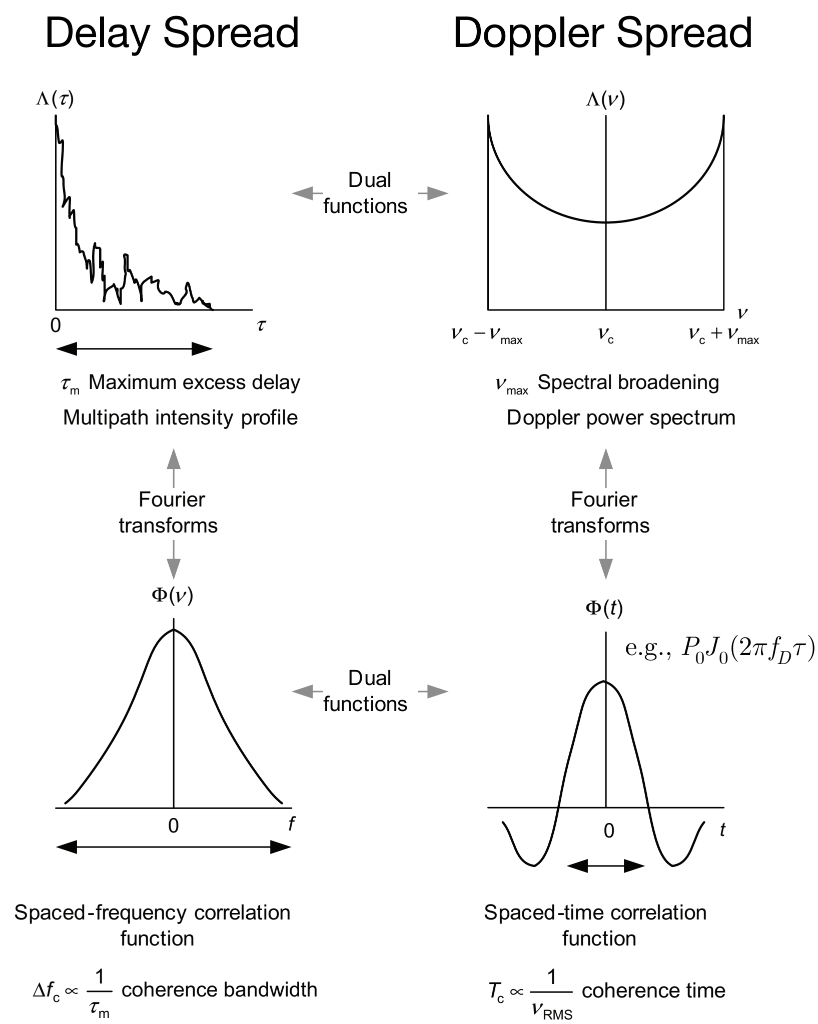
6.7.3 Mobile Radio Propagation Modeling

- The mobile radio channel is more than additive white Gaussian noise being summed with the received signal

- Two types of fading are commonly present:

  1. Signal strength fluctuations known as time selective fading, which is due to motion of the mobile (and to some extent other nearby objects), which in the frequency domain is known as *Doppler spreading*

  2. Fading due to multipath which introduces *delay spread* (or dispersion) into the signal, which in the frequency domain is seen as frequency selective fading

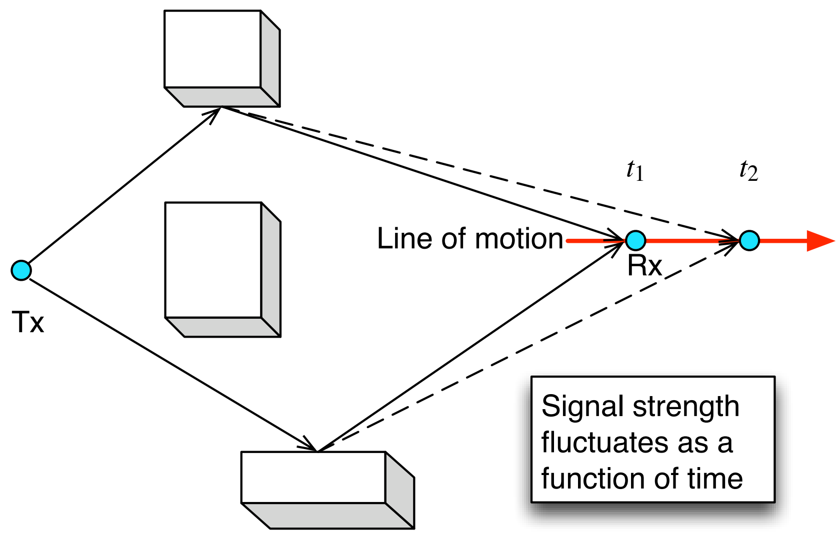


Frequency and Time Selective Fading

Doppler Spreading Details

When the receiver is moving relative to the transmitter, or even when a reflecting surface that forms part of the channel is moving, a change in the carrier frequency known as *Doppler shift* results.

- Recall the Rayleigh fading scenario of notes Chapter 5

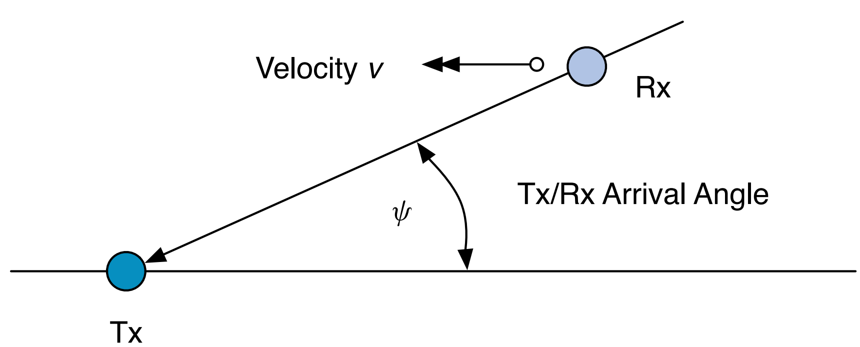


Rayleigh fading scenario

- Consider transmitted signal $Ae^{j2\pi f_0 t}$ being received along the x-axis where x is the position of the receiver relative the transmitter

$$\tilde{r}(t, x) = A(x)e^{j2\pi f_0(t-x/c)}$$

where $A(x)$ is the amplitude of the received signal (path loss) and c is the speed of light



Doppler created by mobile

- For a receiver moving with constant velocity we have position

x being given by

$$x(t) = x_0 + vt$$

with x_0 the initial position at $t = 0$

- With $x(t)$ now a function of time we can write

$$\begin{aligned}\tilde{x}(t) &= A(x_0 + vt)\exp\left[j2\pi f_0\left(t\frac{x_0 + vt}{c}\right)\right] \\ &= A(x_0 + vt)\underbrace{\exp\left[-j2\pi f_0 x_0/c\right]}_{\text{phase}}\underbrace{\exp\left[j2\pi f_0(1 - \frac{v}{c})t\right]}_{\text{frequency}}\end{aligned}$$

- As a result of the motion the effective received carrier frequency is now

$$f_r = f_0(1 - v/c)$$

- The *Doppler shift* is the difference between the transmit and receive frequencies

$$f_D = f_r - f_0 = -f_0 \cdot \frac{v}{c}$$

- In general the Doppler shift may be positive or negative, positive if the objects are approaching and negative if they are departing

- Writing the Doppler shift as $\pm f_0 v/c$ represents the extreme values for a given velocity, while in general we can write

$$f_D = \frac{f_0}{c}\cos(\psi)$$

where ψ is the arrival angle as shown below

Example 6.11: Typical Mobile Doppler Shifts

In this example we will calculate the maximum Doppler shift for a receiver (pedestrian/vehicle) traveling at various velocities. Assume that $f_c = 900$ and 1900 MHz. Note: 1 mph = 0.44704 m/s.

Doppler frequencies

	Frequency, f_c	
Velocity	900 MHz	1900 MHz
$v = 10$ kph (6.2 mph)	8.33 Hz	17.59 Hz
$v = 50$ kph (31.1 mph)	41.67 Hz	87.96 Hz
$v = 100$ kph (62.1 mph)	83.33 Hz	175.93 Hz

Fast Fading: A mobile terminal, e.g., a vehicle in motion, experiences rapid variation in signal strength known as *fast fading*. In the previous section only stationary receivers were considered, so the statistics we considered were a *snap-shot* of what happens over time in the case of fast fading.

- As the mobile terminal moves through the environment the received power fluctuates smoothly over the range of values expected for Rayleigh fading (assuming no line-of-sight path)

- The motion of the receiver is modeled as

$$\tilde{E}(t) = \sum_{n=1}^{N} E_n e^{j(2\pi f_n t + \theta_n)}$$

  where $\tilde{E}(t)$ is the received electric field strength and f_n represents the Doppler shift of a particular ray in the neighborhood of time t

- Using the *Clarke model*[4] the rays arrive at angles $\{\psi_n\}$ uniformly distributed in the horizontal direction

- Note: $f_n = f_D \cos\psi_n$, f_D is the maximum Doppler shift

- The time dependency of the fading can be studied via the complex envelope auto correlation function

$$R_E(\tau) = E\big[\tilde{E}(t)\tilde{E}^*(t+\tau)\big]$$

- We assume that the individual terms of E_n are uncorrelated, but when weighted with the phase that incorporates the Doppler effects we find that

$$\begin{aligned} R_E(\tau) &= E\left[\sum_{n=1}^{N} E_n e^{j(2\pi f_n t + \theta_n)} \sum_{m=1}^{N} E_m^* e^{-j(2\pi f_m (t+\tau) + \theta_m)}\right] \\ &= \sum_{n=1}^{N} E\big[|E_n|^2\big] E\left[e^{-j2\pi f_n \tau}\right] = P_0 E\big[e^{-2\pi f_D \tau \cos\psi}\big] \end{aligned}$$

- The final expectation can be evaluated using the fact that the resulting multipath phase is uniformly distributed on $[-\pi, \pi]$, so finally

$$\begin{aligned} R_E(\tau) &= P_0 E\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j2\pi f_D \tau \cos\psi}\, d\psi\right] \\ &= P_0\, J_0(2\pi f_D \tau) \end{aligned}$$

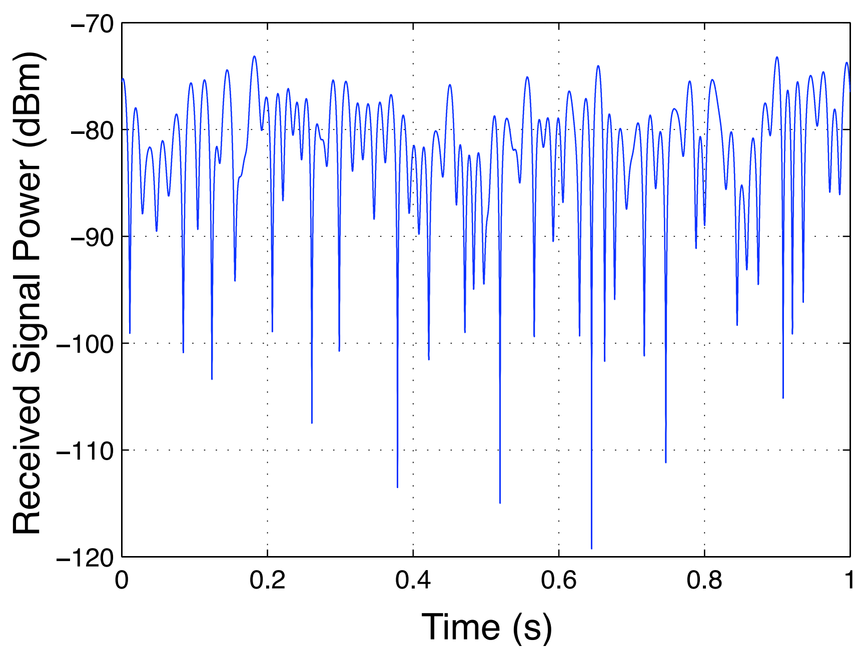
where $J_0(x)$ is a *Bessel function* of the first kind order zero

[4]R.H. Clarke, “A Statistical Theory of Mobile radio Reception,” *Bell System Technical Journal* 47 (July–August 1968, pp. 957–1000.

- The spectrum of the fading process can be obtained by Fourier transforming the autocorrelation function

$$\begin{aligned} S_E(f) &= \mathcal{F}\big[R_E(\tau)\big] \\ &= \mathcal{F}\big[P_0 j_0(2\pi f_D \tau)\big] \\ &= \begin{cases} \frac{P_0}{\sqrt{1-(f/f_D)^2}}, & |f| < f_D \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

- The fading produced by this Doppler spread model for $f_c = 881$ MHz and $v = 50$ kph is shown below:



- As the Doppler frequency goes up the mean fade period goes down and vise-versa

Delay Spreading Details

With a purely frequency selective channel we have a channel impulse response of the form

$$h(t, \tau) = \sum_{l=1}^{L} \tilde{\gamma}_l \delta(\tau - \tau_l)$$

with the complex gains $\tilde{\gamma}_l$ assumed to be constants for now and the path relative time delays, τ_l also assumed to constant.

- Complex baseband signal at the channel output is thus of the form
$$x(t) = \sum_{l=1}^{L} \tilde{\gamma}_l s(t - \tau_l)$$
- The channel is now time invariant, but the frequency response is no longer flat
- Since the channel is fixed or time invariant with respect to t, the channel is *time-flat*
- The power delay profile for a delay spread channel characterizes the spread of received power over a relative or excess time delay interval
- **Note:** OFDM mitigates delay spread by first lowering the symbol rate via demultiplexing and then making use of the cyclic prefix (CP)

6.7.4 Multiple-Input Multiple-Output (MIMO) Systems

- A means to protect against fading by using multiple propagation paths between the transmitter and receiver to advantage
- Space-Time Block Coding and the Alamouti approach (see Z& T p. 551–552 for details)
  - In general multiple transmit and receive antennas are employed
  - The Alamouti approach assumes two inputs (Tx antennas) and one output (Rx antenna)
  - On say even symbol periods we transmit s_0 (ant 1) and s_1 (ant 2)
  - On odd symbols periods we transmit $-s_1^*$ (ant 1) and s_0^* (ant 2)
  - Assuming complex path gains of h_0 and h_1, respectively, the received signals over even and odd symbols are

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned}$$

or in matrix form

$$\mathbf{r} = \mathbf{H}_a \mathbf{s} + \mathbf{n}$$

where $\mathbf{r} = [r_0\ r_1^*]^T$, $\mathbf{s} = [s_0\ s_1]^T$, $\mathbf{n} = [n_0\ n_1^*]^T$, and

$$\mathbf{H}_a = \begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix}$$

– To estimate $\mathbf{s}$ we form $\hat{\mathbf{s}} = \mathbf{H}_a^H \mathbf{r}$

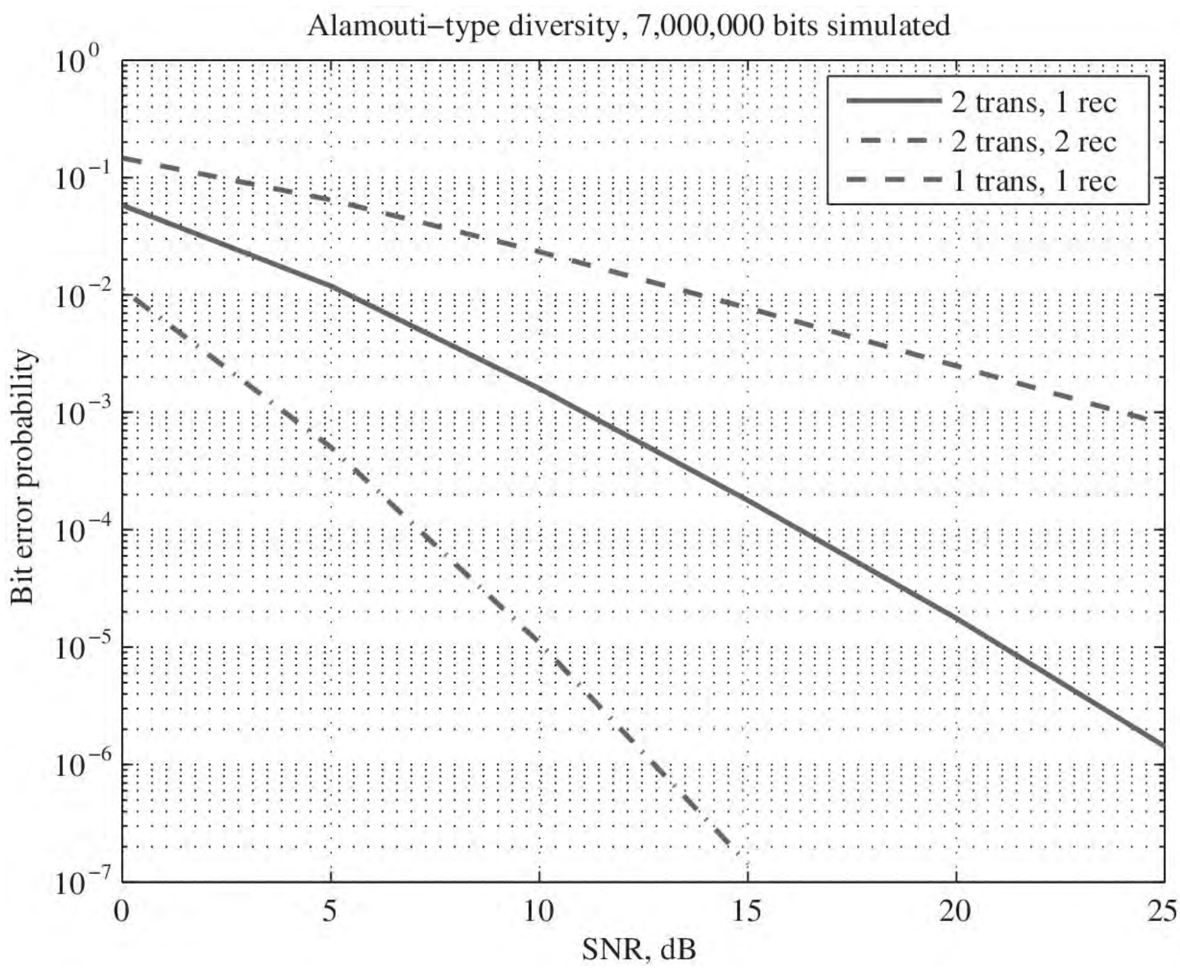
– The symbol estimates are

$$\hat{s}_0 = \big(|h_0|^2 + |h_1|^2\big)s_0 + \tilde{n}_0$$
$$\hat{s}_1 = \big(|h_0|^2 + |h_1|^2\big)s_1 + \tilde{n}_1$$

with $\tilde{n}_0 = h_0^* n_0 + h_1 n_1^*$ and $\tilde{n}_0 = h_1^* n_0 + h_0 n_1^*$

– Maximum likelihood sequence estimation (a minimum Eulcidean distance algorithm) is required to recover the transmitted symbols

– Performance of BPSK in Rayleigh fading is shown below



Alamouti-type diversity space-time coding improvement