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Probabilistic Energy Optimization in Wireless Sensor Networks with Variable Size Griding

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***Abstract*—Due to limited energy supplies, reducing power consumption is an important goal in wireless sensor networks. Clustering techniques are used to reduce power consumption and prolong network lifetime in many existing research efforts, among which grid-based ones are often used due to their simplicity and scalability. However, most existing work uses average distance as a simplification in calculating distance-related power consumption, which leads to a large underestimation of the actual energy depletion rate. In this paper, we propose an energy optimization model based on probabilistic distance distributions, which captures the distance-induced power consumption with high accuracy. We further analyze the uneven traffic distribution in wireless sensor networks and propose a nonuniform griding scheme to balance the energy depletion in all grids. Through our analysis, we are able to obtain the optimal grid size ratio that minimizes the energy consumption. Analytical results are validated through simulation, which shows the promising potentials of our method and the nonuniform griding technique.**

***Index Terms*—Wireless sensor networks, grid-based clustering, distance distribution, energy consumption, variable size grids**

I. Introduction

The miniaturization of wireless sensor nodes has made large-volume commercial production and large-scale real-world deployment possible and viable. Typically, sensor nodes are densely deployed in sensing fields and then left unattended to continuously monitor and report environmental parameters or events; redundant sensor nodes can be put into a sleep mode to conserve energy. Even for nodes with energy harvesting capabilities, energy conservation is an important issue.

Among the sensor node circuitry, the transceiver power amplifier is a major source of energy consumption. To improve the energy efficiency of wireless sensor networks, many research efforts have appeared [1], [2]. In particular, grid-based clustering and routing schemes, where clusters are equal-sized grids in a two-dimensional plane, have become a focus of research due to their simplicity, scalability and feasibility, especially with the aid of GPS and other localization techniques. Once the grid structure is established, nodes can communicate with their cluster head locally, and transmit data to the processing center, or sink node, through neighbor grids.

Various griding schemes have been proposed, but so far most existing work has used the average distance within a grid or between neighbor grids to calculate energy consumption. However, we found that this approach is not accurate and largely underestimates the real value due to the superlinear path loss exponent of over-the-air wireless transmissions [5]. Further, the data flows in sensor networks often follow a many-to-one pattern, making the traffic density highly skewed: nodes around the sink node usually become the bottleneck and will deplete their energy much faster. Since the grid structure will dominate the communication distance between sensor nodes, the following two issues are very important: (i) to build a model that can characterize the node distance more accurately, and (ii) to determine an optimal griding scheme that can balance the energy consumption and prolong network lifetime. These two issues are the main focus of this paper.

In this paper, we propose, analyze and evaluate the energy consumption models in grid-based clustering and routing schemes using the probabilistic distance distributions for sensor nodes at different locations, and introduce the concept of nonuniform griding with *grid size ratio*. Through our analysis, we obtain the optimal grid size ratio of nonuniform griding schemes that can minimize the energy consumption. The remainder of this paper is structured as follows. In Section II, we briefly overview the grid-based clustering schemes and related work. In Section III and IV, we present the energy consumption models with probabilistic distance distributions for nonuniform grid-based sensor networks, followed by numerical and simulation results in Section V. Section VI offers further discussion and concluding remarks.

II. Background and Related Work

Clustering schemes have been an effective way of organizing sensor networks by partitioning sensor nodes into a number of small clusters. Grid-based clustering in particular, as in our previous work [5], has good potential for efficient topology control due to its simplicity and efficiency.

A. Hierarchical Clustering Schemes

With a cluster head (CH) as the coordinator in each cluster, clustering schemes organize the network in a hierarchical way. Redundant nodes can be put into a sleep mode, and the multi-hop forwarding between CHs can avoid long-range, over-the-air transmissions. One general simplification in sensor network analysis is the use of average distance to calculate energy consumption [10], [11]. This largely underestimates the distance distribution between sensor nodes, and the error will become larger due to the superlinear path loss exponent [5].

B. Grid-based Clustering

Grid-based clustering partitions the network into equal-sized square grids, and all nodes in the same grid are equivalent from the routing perspective. Once the sensor nodes have collected any data, they can transmit the data without the

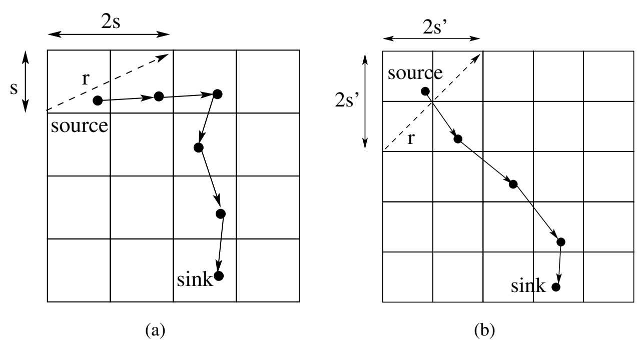

Fig. 1. Manhattan Walk and Diagonal-First Routing.

need to explicitly set up a routing path. [4] and [5] examined two routing schemes, Diagonal-First routing and Manhattan Walk, where any node can communicate via a single-hop transmission with other nodes in an adjacent grid. In Fig. 1 (a) Manhattan Walk, the grid size s is chosen such that any two nodes in horizontally or vertically adjacent grids are within the transmission range (r) of each other, i.e., $s \leq r/\sqrt{5}$. Diagonal-First routing in Fig. 1 (b) further allows the nodes in diagonal grids to be able to communicate directly, i.e., $s' \leq r/\sqrt{8}$. With either griding structure, full coverage of the entire network can be guaranteed, and the modeling and calculation of distance distributions can be greatly facilitated by using the geometric properties of grid-based clustering.

C. Nonuniform Clustering

In a sensor network, the closer a node is to the sink, the more traffic it has to relay. Nonuniform clustering can avoid an early breakdown of the network: grids closer to the sink have smaller sizes, and thus have shorter transmission distances. [12] proposed an unequal clustering model in multi-hop networks, with CHs deterministically deployed. [7] and [8] deduced the relationship between the optimal radio range and the traffic load distribution in a *linear* network. [13] formulated the optimal cluster size as a signomial optimization, with variable-sized disks as clusters, which cannot guarantee full network connectivity.

Inspired by the variable-size griding in image processing [14] and based on our previous work [5], we formulate a probabilistic energy model that can optimize the grid size ratio in order to prolong network lifetime. While [5] builds the models of distance distributions in a two-dimensional network, this paper derives the optimal grid size ratio through further analysis. The simulation results also show the promising potentials of this technique.

III. Probabilistic Distance Models

In this section we derive the probabilistic distance distributions for random sensor networks, which facilitate the distance calculation and can yield results with high accuracy.

A. Coordinate Distributions

Since the election and rotation of cluster heads based on their remaining energy supply can make all sensor nodes in a grid statistically equivalent [4], our analysis focuses on the energy consumption of cluster heads. Given two nodes with coordinates (X_1, Y_1) and (X_2, Y_2), we want to obtain the distribution of node distance $P(D \leq d)$, where $D = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$. With a grid-based clustering scheme, there are two cases of cluster head communications: between cluster heads in either diagonally adjacent grids, or parallel grids, such as PQ and RS in Fig. 2.

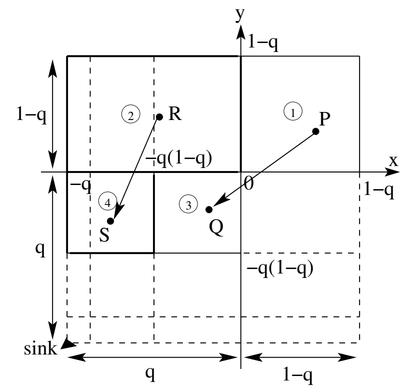

Fig. 2. Nonuniform Griding with Size Ratio q.

Suppose the sink node is located at the lower left corner of the sensing field. Instead of dividing the field into equal sized squares as in Fig. 1, we divide it recursively with size ratio q, a value between 0 and 1 as in Fig. 2. If the sink is located at an arbitrary location in the field, this division can be repeated in the four adjacent quadrants with the sink node as the origin. This nonuniform griding with a fixed ratio q is a special case of variable-size griding.

1) Two Random Nodes in Diagonal Neighbor Grids: We normalize the $A \times A$ sensing field to a unit-size square as in Fig. 2. Grid ① and ③ are diagonal squares with size $(1-q)^2$ and $q^2(1-q)^2$. For P and Q, their coordinate distributions can be formulated with the Heaviside Step Function $H(x)$:

$$\begin{cases} f_{X_1}(x) = f_{Y_1}(x) = \frac{H(x) - H[x-(1-q)]}{1-q} \\ f_{X_2}(x) = f_{Y_2}(x) = \frac{H[x+q(1-q)] - H(x)}{q(1-q)} \end{cases}, \tag{1}$$

where [1]

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases},$$

i.e., $X_1, Y_1 \sim U[0, 1-q]$ and $X_2, Y_2 \sim U[-q(1-q), 0]$.

2) Two Random Nodes in Parallel Neighbor Grids: Grid ② and ④ in Fig. 2 are parallel rectangles with size $(1-q) \times q$ and $q(1-q) \times q^2$. For nodes in these parallel rectangles, e.g., R, the coordinate distribution can be formulated as

$$\begin{cases} f_{X_1}(x) = \frac{H(x+q) - H(x)}{q} \\ f_{Y_1}(x) = \frac{H(x) - H[x-(1-q)]}{1-q} \end{cases}, \tag{2}$$

and for S:

$$\begin{cases} f_{X_2}(x) = \frac{H(x+q) - H(x+q(1-q))}{q^2} \\ f_{Y_2}(x) = \frac{H[x+q(1-q)] - H(x)}{q(1-q)} \end{cases}, \tag{3}$$

[1] Here we assume $H(x) \equiv H_1(x)$ for notation convenience.

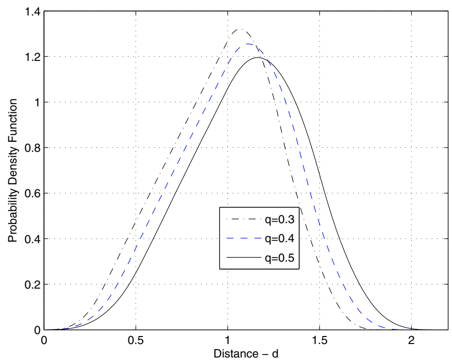

Fig. 3. Probability Density Function $f_{D_{\text{Diag}}}(d)$.

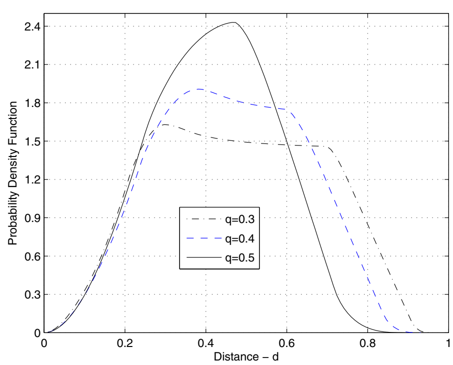

Fig. 4. Probability Density Function $f_{D_{\text{Par}}}(d)$.

i.e., $X_1 \sim U[-q, 0]$, $Y_1 \sim U[0, 1-q]$, and $X_2 \sim U[-q, -q(1-q)]$, $Y_2 \sim U[-q(1-q), 0]$. As in [5], such formulation can be extended to grids of different sizes.

B. Distance Distributions

Let $V = X_1 - X_2$ (or $Y_1 - Y_2$), $S = V^2$, $Z = S_X + S_Y$ and $D = \sqrt{Z}$, our goal is to obtain $f_D(d)$, i.e., the distance probability density function of the two cases above. The detailed four-step derivation of $f_D(d)$ is in [5]. With the same approach, we get the probability density function $f_{D_{\text{Diag}}}(d)$ for the *diagonal* node distance distribution (the case when $(\sqrt{2}-1) \le q \le 1/\sqrt{2}$ is listed as (4)). Figure 3 shows $f_{D_{\text{Diag}}}(d)$ in three cases, where the distances have a range of $[0, \sqrt{2}(1+q)]$. The curves have a tilted bell shape because the squares diagonal to each other are not of the same size.

$f_{D_{\text{Par}}}(d)$ for the *parallel* node distance distribution, when $(1 - 1/\sqrt{2}) \le q \le \sqrt{2}/3$, is listed in our technical report [9]. Figure 4 shows three cases of $f_{D_{\text{Par}}}(d)$, in which the distance range is $[0, \sqrt{(1-q^2)^2 + q^2}]$. As q gets larger, $f_{D_{\text{Par}}}(d)$ gradually changes from a shape similar to a uniform distribution to a tilted bell shape. Apparently, neither $f_{D_{\text{Diag}}}(d)$ nor $f_{D_{\text{Par}}}(d)$ can be simply approximated by a Gaussian or uniform distribution.

IV. Probabilistic Energy Optimization

Energy consumption is an important performance metric for wireless sensor networks, which largely depends on the distance between transceivers. In this section we present our analysis based on the distance distribution derived in the last section.

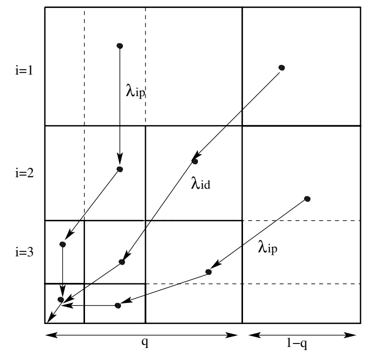

Fig. 5. Many-To-One Traffic Pattern.

A. Wireless Channel and Traffic Model

1) Energy Consumption Between Transceivers: According to [6], the energy consumed by the radio transmitter is

$$E_{\text{Tx}} = \lambda\epsilon \int x^\alpha f_D(x)dx, \quad (5)$$

where λ is data transmission rate, ϵ is a constant related to the environment, and α is path loss exponent. $f_D(x)$ can be $f_{D_{\text{Diag}}}(d)$ or $f_{D_{\text{Par}}}(d)$, depending on node location. Although (5) assumes perfect power control, the result can provide a useful performance bound.

2) Accumulated Many-To-One Traffic: As in Section II-C, the multi-hop forwarding in wireless sensor networks leads to the many-to-one traffic pattern. In Fig. 5 where the Diagonal-First routing is used, the CHs in the i-th ring receive the traffic originating from its own cluster, and the traffic relayed from the $(i-1)$-th ring, and then they forward the combined traffic to the $(i+1)$-th ring. The traffic analysis for the Manhattan-Walk rouing scheme is similar.

B. Energy Optimization

Let E_i be the energy consumed by CHs in the i-th ring:

$$E_i = \lambda_i(E_{\text{Re}} + E_{\text{Te}}) + \lambda_i E_{\text{Tx}}, \quad (6)$$

where λ_i is the data rate going through the i-th ring, including the data rate from the sensors to their CHs, and the data rate between CHs in adjacent rings. E_{Re} and E_{Te} are the energy consumed by the receiver and transmitter circuitry, and E_{Tx} is the one used by the transmitter power amplifier. According to [3] we have $E_{\text{Re}} = E_{\text{Te}} = E_e$, thus,

$$E_i = \lambda_i(2E_e + E_{\text{Tx}}). \quad (7)$$

From Fig. 5, λ_i is essentially the bit rate of aggregate traffic that originates from the grids in the outermost ring through the i-th ring. Since the sensing field is symmetric, the aggregated data can be divided into two parts: λ_{ip} that is between parallel rectangles (in both the upper and lower half of the field), and λ_{id} that is between diagonal squares, i.e., $\lambda_i = 2\lambda_{ip} + \lambda_{id}$. Suppose node density is ρ, then

$$\begin{cases} \lambda_{ip} = \frac{A^2 q}{1+q}(1-q^{2i})\rho\lambda \\ \lambda_{id} = \frac{A^2(1-q)}{1+q}(1-q^{2i})\rho\lambda \end{cases}. \quad (8)$$

$$\begin{cases}
\frac{d^3}{2q^2} & 0 \le d \le q \\
\frac{2d^2}{q} - \frac{d^3}{2q^2} - d & q \le d \le \sqrt{2}q \\
d\sin^{-1}(\frac{d^2-2q^2}{d^2}) + \frac{2d}{q}(d - \sqrt{d^2-q^2}) & \sqrt{2}q \le d \le 1 \\
d\sin^{-1}(\frac{d^2-2q^2}{d^2}) + \frac{2d}{q^2}[(1+q)d - q\sqrt{d^2-q^2}] - \frac{d^3}{q^2} - \frac{d}{q^2} & 1 \le d \le \sqrt{1+q^2} \\
\frac{d}{q}[\sin^{-1}(\frac{d^2-2}{d^2}) + (1+q)\sin^{-1}(\frac{d^2-2q^2}{d^2})] - 2d\frac{\sqrt{d^2-1}}{q} + d & \\
\quad + 2d\frac{1+q}{q^2}(d - \sqrt{d^2-q^2}) & \sqrt{1+q^2} \le d \le \sqrt{2} \\
d\frac{1+q}{q^2}[\sin^{-1}(\frac{d^2-2}{d^2}) + q\sin^{-1}(\frac{d^2-2q^2}{d^2})] + \frac{d^3}{2q^2} + d\frac{1+q^2}{q^2} & \\
\quad + 2d\frac{1+q}{q^2}(d - \sqrt{d^2-q^2} - \sqrt{d^2-1}) & \sqrt{2} \le d \le (1+q) \\
d\frac{1+q}{q^2}[\sin^{-1}(\frac{d^2-2}{d^2}) + q\sin^{-1}(\frac{d^2-2q^2}{d^2})] + \frac{3d^3}{2q^2} + 2d\frac{1+q+q^2}{q^2} & \\
\quad - 2d\frac{1+q}{q^2}(\sqrt{d^2-q^2} + \sqrt{d^2-1}) & (1+q) \le d \le \sqrt{q^2+(1+q)^2} \\
d\frac{1+q}{q^2}[\sin^{-1}(\frac{d^2-2}{d^2}) + q\sin^{-1}(\frac{2(1+q)^2-d^2}{d^2})] + \frac{d^3}{2q^2} + \frac{d}{q^2} & \\
\quad + \frac{2d}{q^2}[q\sqrt{d^2-(1+q)^2} - (1+q)\sqrt{d^2-1}] & \sqrt{q^2+(1+q)^2} \le d \le \sqrt{1+(1+q)^2} \\
d\frac{(1+q)^2}{q^2}\sin^{-1}[\frac{2(1+q)^2-d^2}{d^2}] - \frac{d^3}{2q^2} & \\
\quad - d\frac{1+q}{q^2}[(1+q) - 2\sqrt{d^2-(1+q)^2}] & \sqrt{1+(1+q)^2} \le d \le \sqrt{2}(1+q) \\
0 & \text{otherwise}
\end{cases} \qquad (4)$$

Therefore,

$$E_i = 2\lambda_{ip}\left(2E_e + \epsilon\int x^\alpha f_{D_{\mathrm{Par}}}(x)dx\right) + \lambda_{id}\left(2E_e + \epsilon\int x^\alpha f_{D_{\mathrm{Diag}}}(x)dx\right). \qquad (9)$$

We formulate the problem of minimizing the total network energy consumption as

$$\begin{aligned} & \min \textstyle\sum_{i=1}^K E_i \\ s.t. \quad & Aq^{K-1} > r_0 \\ & Aq^K <= r_0 \end{aligned} \qquad (10)$$

where r_0 is the minimal distance between wireless transceivers, and K is the maximum number of hops from the source node to the sink. Meanwhile, the problem of maximizing network lifetime is equivalent to the following:

$$\min \max_i E_i, \qquad (11)$$

where $i = 1...K^2$, so E_i is calculated for each grid instead of each ring, i.e., minimizing the maximal energy consumption of all grids to keep the CHs alive and maintain the grid structure.

V. Performance Evaluation

In this section, we verify the distance distributions in Section III by simulation, evaluate the network-wide energy consumption, and compare it with average distance model.

A. Distance Verification

Figure 6 shows the cumulative distribution function and simulation results for diagonal grids, when $q = 0.4$ and 0.7, by generating $1,000$ pairs of random points. The simulation results and distribution function match closely. The dashed lines in Fig. 6 are the distribution average (where the value of CDF is 0.5), while the dotted lines are the min-max average, by taking the average of min and max values.

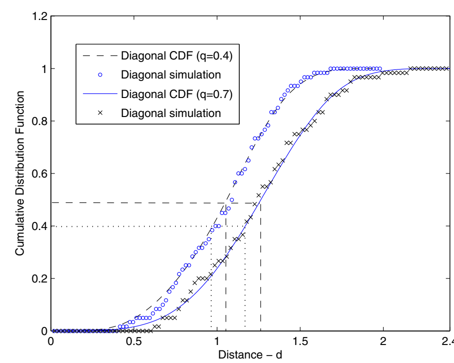

Fig. 6. Numerical and Simulation Results For Distance Distribution.

B. Network Energy Optimization

In order to see the impact of using the average distance and distance distribution approach on the network energy optimization, we simulate a wireless sensor network over an area of $100 \times 100\ m^2$, with $1,000$ sensor nodes.

1) Simulation Setup: Both free-space propagation and multi-path fading models are used in simulation: if the wireless transceiver distance is less than the cross-over distance d_c, then the Friis free-space model is used; otherwise the two-ray ground model will be applied [3]:

$$E_{\mathrm{Tx}} = \begin{cases} k\epsilon_{\mathrm{Friis}} d^2 & d \le d_c \\ k\epsilon_{\mathrm{two-ray}} d^4 & d \ge d_c \end{cases}. \qquad (12)$$

[3] also describes the calculation of $\epsilon_{\mathrm{Friis}}$, $\epsilon_{\mathrm{two-ray}}$, and d_c:

$$d_c = \frac{4\pi\sqrt{L}h_r h_t}{\lambda_c}, \qquad (13)$$

where $L \ge 1$ is the system loss factor, h_r and h_t are the height of the receiver and transmitter antenna, and λ_c is the wavelength of carrier signal. In our simulation, we used $h_t = h_r = 0.6\ m$, no system loss ($L = 1$), $2.4\ GHz$

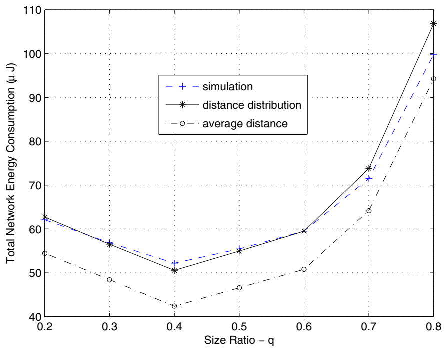

Fig. 7. Total Network Energy Consumption.

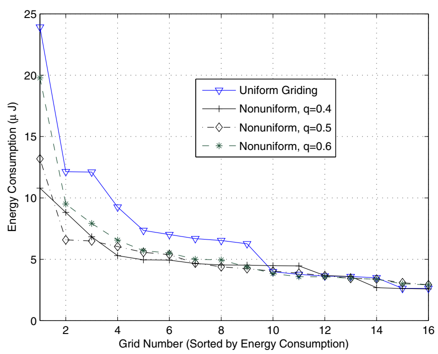

Fig. 8. Per-Grid Energy Consumption with Nonuniform Griding.

radio frequency[2], and $\lambda_c = \frac{3\times10^8}{2.4\times10^9} = 0.125\ m$. Plugging these numbers into (13), we get $d_c = 36.2\ m$. The corresponding coefficients are $\epsilon_{\text{Friis}} = 0.69\ pJ/bit/m^2$ and $\epsilon_{\text{two-ray}} = 0.051\ pJ/bit/m^4$. The energy consumed per bit in the transceiver electronics is $E_e = 50\ nJ/bit$.

2) Simulation Results: In Fig. 7, the energy consumption model using distance distributions matches the simulation results with high accuracy, while the model using average node distance underestimates the real value by as much as 10%. There also exists an optimal size ratio q around 0.4, which minimizes the total network energy consumption. Intuitively, when q is small, the grids far away from the sink will cover a large area, therefore both the transmission range between the CHs in adjacent rings and the traffic that is relayed to the next ring are large. If q increases to a reasonable value, both the transmission range and traffic volume are decreased, and a significant amount of energy can be conserved. If q continues increasing, all the grids will have smaller sizes, more hops are needed to reach the sink node, and many more CHs have to be active at the same time.

Figure 8 shows the balancing effect of energy consumption in nonuniform griding, which is calculated for each grid and sorted in descending order. Compared with the results from uniform griding, it is obvious that the nonuniform griding with a proper grid size ratio can reduce the maximum energy consumption, and the overall energy consumption is more balanced. As a result, nonuniform griding can prolong network lifetime since all CHs have approximately the same residual energy level. Compared to Fig. 7, the optimal value of q is still in the same range, i.e., when $q = 0.4$, the energy consumption per grid is almost uniform.

[2]According to IEEE 802.15.4 specification, ZigBee operates in the Industrial, Scientific and Medical (ISM) radio bands: 868 MHz in Europe, 915 MHz in the USA and Australia, and 2.4 GHz in most other jurisdictions worldwide. Our calculation can be easily applied to other frequency bands.

VI. Further Discussion and Conclusions

Due to simplicity and scalability, grid-based clustering has become an efficient sensor network communication and coordination scheme. In this paper we first propose an energy consumption model based on distance distributions instead of using the average distance. This model can be used in nonuniform grid-based clustering, which makes both the data traffic and energy consumption more balanced, and prolongs network lifetime. We also investigate the impact of nonuniform griding with a grid size ratio through analysis and calculation, where the distance distribution models are used. Our analysis reveals the importance of grid-based clustering schemes and the optimal grid size ratio that can balance the overall energy consumption. The performance evaluation showed the capability of the distance models derived in this paper, and the potential of the nonuniform griding scheme.

Acknowledgment

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